TUTORIAL PAPER IV

SELF-ORGANIZATION IN HUMAN SYSTEMS

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ABSTRACT

In recent years new concepts have emerged for the natural sciences related to the self-organization or structural evolution of complex systems. This new paradigm is discussed, and several applications in the field of economics, and urban systems are described.

INTRODUCTION

In our attempts to understand and 'deal with' the complexities of the world around us we commonly construct 'models' either intuitively in our heads, or explicitly by setting them down on paper. The essence of modelling is that it be a 'reduced description' where 'superfluous' detail and particularity are passed over, and only the essential remains. Briefly then we search for something which is easy enough to work with, but which nevertheless captures and governs what we consider to be the important features.

Yet we often have cause to ask : what level of description is really required to describe the evolution of a particular system ? Often a 'system' analysis or a 'model' of a system is written as a set of interacting differential equations describing the time rate of change of certain 'variables'. These latter are usually aggregate or average 'variables' over the system or some region of it, and their change in time is supposed due to the occurrence of 'typical' processes expressing average behaviour.

The first fundamental point of divergence of common sense with many 'models' is their 'determinism'. The question we must ask is : when is the passage from a complex, complete real system to a reduced, deterministic description adequate ? The answer has recently become clear as a result of progress made in the natural sciences concerning the origin of structure and organization in the universe (Glansdorff and Prigogine, 1971). In systems at or very near to thermodynamic equilibrium the macroscopic, reduced description is valid. However, for systems which are far from equilibrium, open to flows of energy and matter then non-linear interactions can lead to a breakdown of the macroscopic description because of the possibility of bifurcation and of the multiplicity of solutions. Where bifurcation occurs the reduced description is inadequate.

Consider the modelling of the growth of a population X inhabiting a region with limited resources. The basic description is simply to suppose probabilities of birth and death due to the different processes going on in the system, and to construct the so-called 'Master equation' which governs its evolution. This essentially considers the net in or out movement per unit time for each 'slice' of probability P(X,t) and in this way generates both the movement and change in shape of the probability distribution in time.

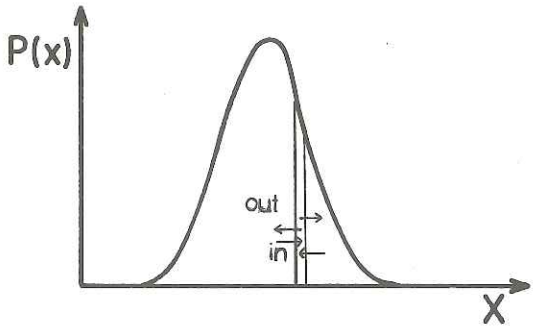


Fig.(1). Probability distribution of X.

Now if we study the change of the average value of X, we must simply multiply the master equation (1) by X and integrate over all X. The 'first moment' approximation as this is known then tells us how the average values of X changes as a result of the various processes which cause the increase or decrease of X. If, for example, we consider a probability of birth per individual per unit time which depends on the quantity of resources left in the system :B(1 + X/N), and the simple probability of death per individual per unit time, M then, we have for the Master equation :

$$\frac{dP}{}(X,t) = B\left[(X-1)\ (1 + X/N)\ P(X-1) - X\ \left(1-\frac{X+1}{N}\right) P(X)\right] + M\ \left[\ (X+1)\ P(X+1)\ -\ XP(X)\right] \qquad (1)$$

and multiplying by X, integrating and considering unit area of the system we find :

$$\frac{dx}{dt} = bx\ \left(1-\frac{x}{N}\right) - mx \qquad (2)$$

the logistic equation, where x is the density of individual, and b and m have dimensions of inverse time, N is the density of resources in absence of x.

Typically, we have a dynamics of the type shown in fig. (2).

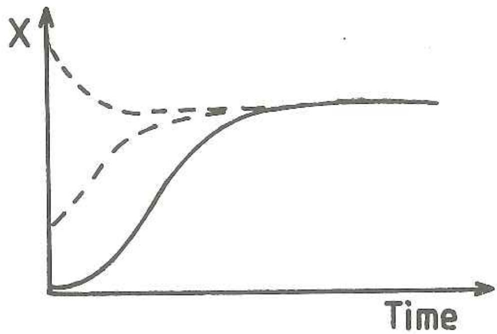


Fig.(2).Dynamic growth or decline according to equation (2).

However, if we examine more carefully this equation, then we find that it affords a very simple example of bifurcation. We note that for example, $x = 0$ is always a solution. That is $\frac{dx}{dt} = 0$ when $x = 0$. However, there may exist a second solution, $x=N(1-m/b)$ corresponding to a real positive solution provided that $b > m$. Thus we may say, that a study of the stationary states of the kinetic equation does not permit to us to say unambiguously what the population of the system is for $b>m$. It could be $N(1-m/b)$ or 0, and at present we have no information which.

However, extra information can be obtained by studying the kinetic equation and thinking about the 'stability' of these two solutions. Since in the real world we know that the actual density of the population x will fluctuate around the average values predicted by the mathematical model (2), then in the real world for a stationary state to persist, it must have the property of 'resisting' or 'damping' such fluctuations.

The stability of the two possible states $x_1 = 0$ and $x_2 = N(1-m/b)$ can be ascertained by considering the behaviour of the kinetic equation for arbitrary values of x. In this way, we have marked on a diagram the direction of motion of x (either increasing, or decreasing) that the kinetic equation obliges for any particular value of x fixed N and m, where $N > 0$ and $m > 0$and for different values of $b > 0$.

This allows us to draw the diagram (3) showing the global stability properties of the system.

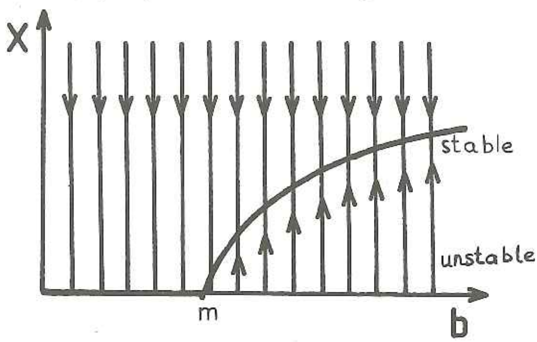


Fig.(3) A diagram showing the behaviour of the system and its global stability properties.

Thus, when $b < m$, there is only one solution, $x_1 = 0$, and this solution is globally stable, attracting all initial values of x. When $b > m$ however, there are two solutions, but $x = 0$ is unstable. The solution $x = N(1-m/b)$ is stable and attracts all initial values of x as shown in fig.(3).

This rather trivial result, can be made considerably more interesting if we write the 'mortality' term m, in terms of the disappearance of x because of a predator, whose density we shall suppose fixed, and which 'grazes' on the prey x when it encounters it. However, when prey is abundant, we will suppose that each predator can only eat his 'fill', and therefore total consumption of x cannot exceed some maximum value per predator. The simplest term which represents such a situation is the following :

$$mx = \frac{SyX}{1+X} \qquad (3)$$

where, when x is small ($x \gg 1$) then the rate of decrease of x is proportional to the rate of encounters between predator and prey (Syx) while if x is large, the rate of consumption of x is simply proportional to the density of predators (Sy).

Our modified equation reads :

$$\frac{dx}{dt} = bx\left(1-\frac{x}{N}\right) - \frac{sxy}{1+x} \tag{4}$$

Again, $x = 0$ is always a solution, but there are possibly two other solutions given by the quadratic equation :

$$x^2 - (N-1)x - N(1-sy/b) = 0 \tag{5}$$

which leads to

$$\begin{aligned} x^- &= \frac{N-1}{2} - \frac{1}{2}\sqrt{(N-1)^2 + 4N(1-sy/b)} \\ x^+ &= \frac{N-1}{2} + \frac{1}{2}\sqrt{(N-1)^2 + 4N(1-sy/b)} \end{aligned} \tag{6}$$

The behavior of the system as a function of b is shown below :

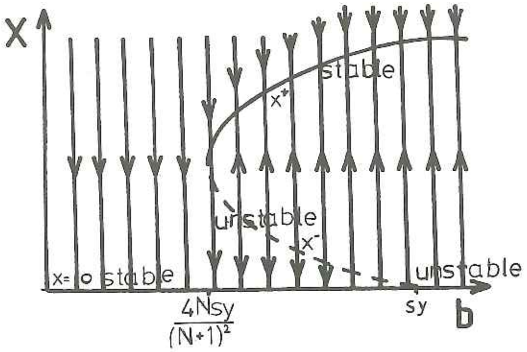


Fig.(4). Solutions x as a function b.

Thus, for $b < \frac{4Nsy}{(N+1)^2}$ we have one stable root $x = 0$.

for $\frac{4Nsy}{(N+1)^2} < b < sy$ then <u>both</u> $x = 0$ and x^+ are stable, through x^- is unstable.

for $b > sy$, we have only a single stable root, x^+.

Here we see that for a certain range of parameter values, we have three simultaneously possible solutions for the deterministic, average equations, and now even stability analysis does not suffice to confine the system to a unique state, since two of there solutions are stable. Thus, in reality, the state which we shall observe for a particular case will depend on its history. This is a first very simple example of how the concepts of 'memory' and of 'choice' enter into a problem described by a set of macroscopic equations.

The importance of this particular model for agriculture, fisheries and any exploitation of a naturally reproducing species can be seen by considering the 'bifurcation' diagram drawn for a fixed b and N, but for increasing numbers of 'grazing' species y.

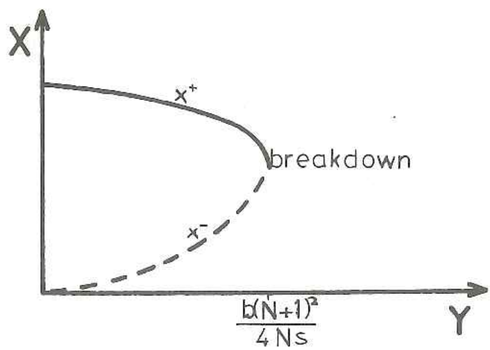


Fig.(5). Possible solutions of x as a function of increasing grazing y.

Thus, increasing the grazing herd steadily,at first leads only to gradual decrease in the density of the prey. However, at a certain point, only a small increase in y, produces a sudden collapse of prey numbers, that will entrain the subsequent dissapearance of the grazing herd and a breakdown in the system.

In reality, the calculation of this critical point must take into account the precise nature of the fluctuations of population, of b of N and of S, for on these will depend the level of grazing which can be maintained without risk. Failure to do this has led to ecological and economic disasters in the past !

In chemistry some very much more complex bifurcations have been studied, giving rise to many different possible solutions of the chemical kinetics, solutions which differ qualitatively. For example, the reaction scheme,

$$A \longrightarrow X$$

$$B + X \longrightarrow Y + D$$

$$2X + Y \longrightarrow 3X$$

$$X \longrightarrow E$$

where X produces Y, which in turn procedure X has been intensively studied by the Brussel's school (it is even known as the Brusselator) and various different types of self-organization have been found. By regulating the flows of the initial and final products. A, B, D and E, one can move away from equilibrium and, at a certain critical distance an instability occurs. This threshold marks the point at which the least fluctuation can cause the system to leave its uniform stationary state.

When this occurs a fluctuation is amplified and drives the system to some state characterized by the coherent behaviour of an incredible number of molecules, forming perhaps a moving zone of high concentration of component - a chemical wave - wherein the chemical reactions maintain the spatial organization.

These are 'dissipative structures' new, organized states of matter some examples of which are shown in the figures (6) and (7). They correspond to organizations of the system which exceed by many magnitudes the scale of the interactions between the individual elements, in this case molecular forces, in fact over lengths related to the non-linearities and the diffusive forces in the system. All that is required by a structure to 'explain' its persistence once it has arisen stochastically from an instability, is that it be stable.

This description contains both deterministic mechanisms (the chemical equations) and stochastic, random effects (the fluctuations) and it is these latter that are of particular importance when the system is near to points at which a new organization may change. These points are called bifurcation points.

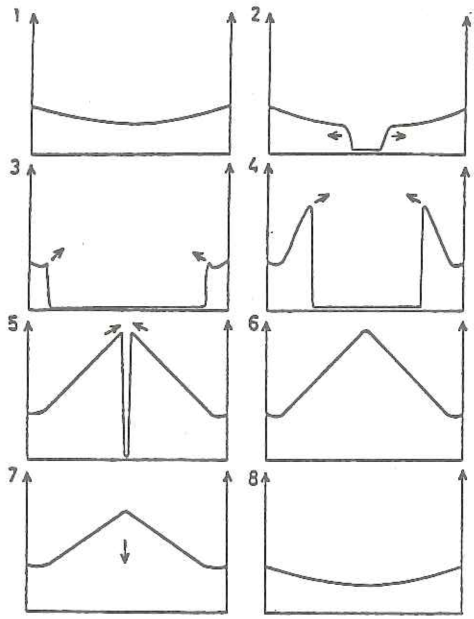


Fig.(6). A cyclic spatio-temporal stucture of propagation : chemical 'waves', as the concentration of intermediate X follows the sequence indicated in 1⟶8.

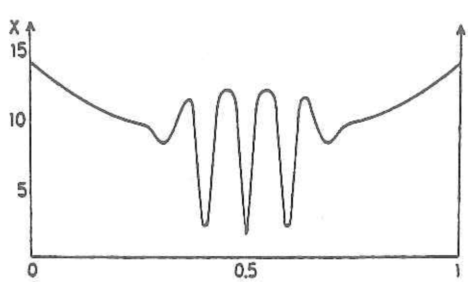


Fig.(7). If we allow for the diffusion of A into the system from the walls, then we can have the formation of a dissipative structure having its own length scale.

Complex systems can ofcourse have a whole series of bifurcations points, as for example we shown in fig.(8), where the diagram of possible solutions is drawn as a function of some parameter p involved in the interactions.

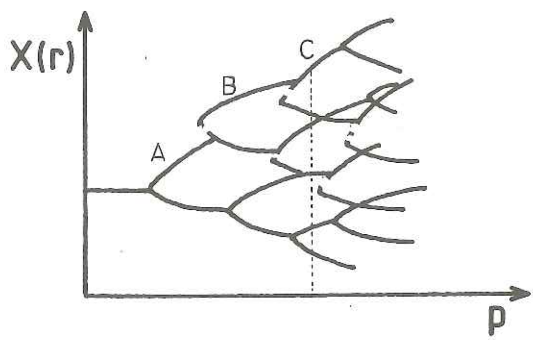


Fig.(8). A bifurcation diagram showing the possible solutions as a function of parameter p involved in the interaction.

Between two bifurcation points, the system follows deterministic laws (such as those of chemical kinetics) but near the point of bifurcation it is the fluctuations which play an essential role in determining the branch that the system chooses. Such a point of view introduces the concept of 'history' into the explanation of the state of the systems. For example, in fig.(8), the 'explanation' of the fact that the system is organized according to the solution

C, necessarily refers to the passage through the structures B and A. No 'explanation' can ever deduce the unique necessity of finding the system in state C for the particular value of the parameter P.

A vital point that must be understood is that these two different aspects of evolution correspond to situations 'far from', or 'near to' a bifurcation point. In reality, what we have is that far from a bifurcation point, the probability distribution is sharp and singly humped. Thus the 'average' or macroscopic equations of our model (the first moment approximation) effectively govern and determine the state of the system. However, near a bifurcation point, the non-linearities on the interactions cause the probability distribution of the underlying stochastic process to kink, and lead to a double humped distribution.

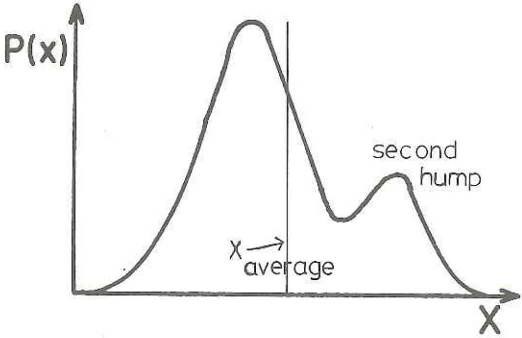


Fig.(9). Appearance of a second hump due to non-linear interactions.

Thus, the first moment or average description is now hopelessly inadequate to describe what the system will now do, and in fact what occurs is that the system may jump to one or other of the humps. An important remark concerns the 'danger' of interpreting the behaviour of a system as being governed by a potential function. For example, one could, retrospectively construct a 'potential' which behaved as some inverse probability func tion,

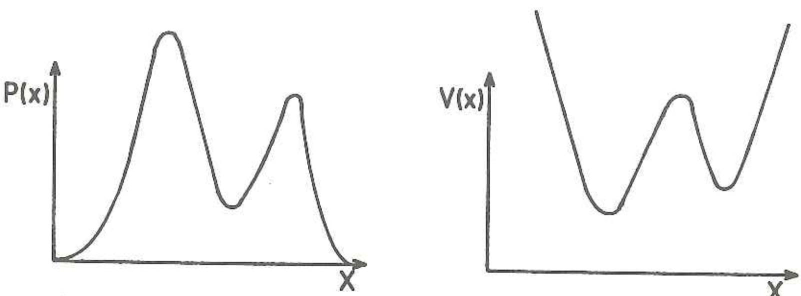


Fig.(10).

a) Double humped probability distribution

b) Potential well designed to 'mimic' (a).

but in general such a construction serves no purpose. Firstly, for more than one variable it is in general impossible to construct such a function, meaning that the class of non-linear dynamical systems contains as a small subset thos derived from a potential function. Secondly, we must recall that the differential equations of our 'model' are in fact only the first moment approximations to the master equation and that it is this latter equation that really governs the system. Thus, even for a one variable problem where it is possible to intergrate the right-hand side of the differential equation in order to obtain a potential function, there is no guarantee that this will <u>in fact</u> imitate correctly the true shape of the probability distribution. For example, it is true that we can integrate, the logistic equation $\frac{dx}{dt} = ax - bx^2$ to give a 'potential', ($V(x) = a\frac{x^2}{2} - b\frac{x^3}{3}$), but in reality, we could have different stochastic dynamics underlying the first moment approximation. For example, the presence of term :

$$X \longrightarrow X + 1 \quad \text{with probability } \frac{AX^2}{2}$$

and

$$X \longrightarrow X - 1 \quad \text{with probability } \frac{AX^2}{2}$$

does not change the 'average' equation, but does change the 'shape' of the probability distribution $P(X,t)$, and hence the 'potentials' obtained by integrating the first moment equation would mimic incorrectly the behaviour of the system when perturbed from its stationary state. In a word then, the clean, closed, geometrical world of potentials and catastrophe theory is in general not relevent to the real, somewhat messy but much more interesting, world of complex systems evolving through successive structural instabilities.

This type of evolution, involving both determinism and chance, has been called 'order by fluctuation', (Nicolis and Prigogine, 1977) and we see that this extension of the physical sciences offers us a paradigm which is potentially of great importance for the biological and social sciences (Prigogine, Allen and Herman, 1977). There is already an extensive literature concerning different applications of these new ideas in various domains. In the study of oscillatory biological phenomena (Goldbeter and Caplan, 1976), (Goldbeter and Nicolis, 1976), for example, and in the problem of 'morphogenesis' (Erneux and Hiernaux, 1979) in the early states of embryo development. Also, the development of models treating the problem of cancerous growth, as resulting from an instability of the immune system, have been made and explored in both steady and 'noisy' environments (Lefever and Horsthemke, 1979). Other studies have been undertaken which explore the role of these new ideas in our understanding of the 'order' that reigns within animal population, and in particular, in colonies of social insects. (Deneubourg and Allen, 1976), (Deneubourg, 1976). We shall not go further into such questions here, but move on to discuss the impact of these new ideas on our understanding of human systems.

2. SELF-ORGANIZATION IN A 'SIMPLE MARKET SYSTEM.

Having mentioned the evolutionary paradigm offered by 'dissipative structures' in the realms of chemistry, biology and ecology we now turn to a brief description of some recent applications to human systems. The characteristic of such systems is that they are made up of a multitude of 'actors' of different types, each having its own particular criteria and values, as well as different opportunities and power in the systems.

The economic and social sciences have developed in an attempt to understand and clarify the workings of society, and where mathematical modelling has been used, it has been largely based on the analogy between equilibrium physics (for example, an equation of state $P = RT/V$). Thus a 'global utility' for social or economic system is often used to 'explain' its evolution, in a manner akin to the 'increase' of entropy characteristic of an isolated physical system. Similarly, some polynomial form is often supposed to express this 'global utility' or potential and since this can lead to multiple solutions,to folds and cusps, the evolution of the system as a whole is 'pictured' as being one of movement along this surface, and of 'catastrophic' jumps between them. It is our contention that for most complex systems, particularly humain containing ones, this analogy is in general false, and that the 'construction' of such a potential can only be performed 'after the event' and as we have discussed above only constructed to 'mimic' what has been observed and which in fact results from the complex dynamic interplay of the decisions of different actors.

What we are concerned with throughout this paper, is what Herbert Simon has called the ineradicable scandal of Economic Theory - imperfect competition. In reality this scandal extends over the whole of social science and biology and concerns, the dynamic interaction of what can only loose be called 'supply' and 'demand'. It concerns emergence of structure and organizations in open systems.

In order to proceed further, we must now try to identify the significant actors of the system, whose decisions, and the interplay of these, will result in the particular patterns of consumption observed. These we shall suppose to be consumers of varying wealth and taste, and entrepreneurs investing in the production of particular products, and adopting different possible strategies to secure and make profitable this investment.

The next phase of modelling is to attempt to construct the interaction mechanisms of these actors, which in principle requires a knowledge of their values and preferences, and ofcourse how these values conflict and reinforce each other as the system evolves.

How can several different criteria be 'combined' in order to given a measure of the probability of a particular decision ?(Roubens, 1980) The basic idea accepted by multicriteria analytists is that we may suppose that a given actor is at least conscious of some major criteria, in each of which he can define a direction of preferençe, and also that in addition, he can assign some measure of their relative importance, even if it is very vague. Clearly, the idea of a 'pay-off' which will occur in the future following an action involves the actors capacity to believe that he can predict the future over such a time. Thus it depends on his confidence in the 'model' he is using. This is yet another aspect of 'learning', which as we shall see permeates the discussion of modelling in human systems.

Another vitally important factor which must be included in any modelling of decisions is the fact that we have in general non-linear responses to given changes in stimuli. In general then the problem of assigning a number to a given value of a criterion such that it measures our 'reaction' and 'sensitivity' to that reading, comes down to some non-linear projection. However, such an approach will also open the door to the consideration of qualitative factors, for in reality there is no difference between the input of a 'quantity' to which we may be unduly sensitive, and a 'quality' which although the input is not strictly a number, nevertheless may have a number assigned to it.

A particularly clear way of visualizing the problem is to suppose that the axes corresponding to each criterion have a common origin, which represents the 'ideal', the most preferable solutions imaginable. For example, we may wish for the biggest, fastest, most comfortable car, butwhich costs zero money ! If this is the origin of our value space, then in fact the choices open to us will be out away from this origin, offering various compromises of size, speed, comfort and price. The question is, how are these different possibilities perceived and weighed by individuals?

Let us suppose that we consider two choices only, and that the axes are viewed as having importance (α_1, α_2, α_3).

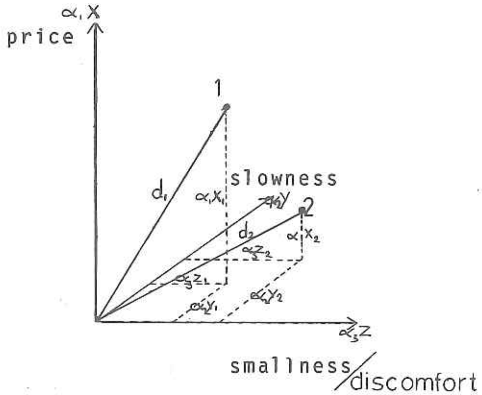


Fig.(11)

The two choices viewed by this individual are now some distance from the origin, and in general we see that these 'distances' should be calculated using the formula,

$$d_1 = \sqrt{(\alpha_1 x_1)^2 + (\alpha_2 y_1)^2 + (\alpha_3 z_1)^2}$$

$$d_2 = \sqrt{(\alpha_1 x_2)^2 + (\alpha_2 y_2)^2 + (\alpha_3 z_2)^2}$$

Thus if α_1, is increased the probability of choosing the cheaper car is increased. In this simple example then, we have supposed a linear sensitivity to each 'pay-off' value, given by the weights as well as complete certainty as to the values of the pay-offs. In reality, we must admit the possibility that instead of simply using a weighting we should use a 'projector' which will map the 'pay-off' onto each axis or criterion, taking into account the effects of constraints which may lead to thresholds and extreme sensitivity in certain ranges. Clearly, a Boolean type analysis is an extreme example of such non-linearity, and indeed corresponds to a 'satisficing' behaviour which may be important, particularly in uncertain situations.

Another point which should be emphasized is that when ever we examine a decisional problem involving a given set of choices, say which car should we buy for example, it is important to remember that in addition to the given choices of different makes of car, there are the 'other' choices lying outside the 'automobile market'. Thus, in the example, we have considered above, we must add to the two choices we have shown, by considering the 'pay-off' associated with not buying a car. It costs less, probably, but is slower and less comfortable. Thus our diagram should be amended to look as below.

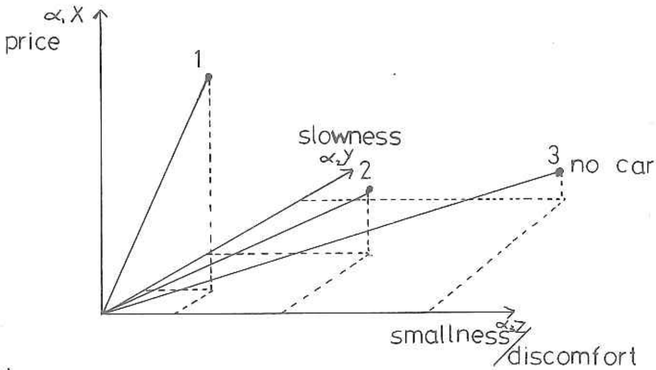


Fig.(12)

The presence of this third choice means that clearly there is some probability that is will be chosen, and what we may note is that as the 'distances' d_1 and d_2 both increases for the 'poor' individual, so the probability of making the third choice increase. This effect will be all the more strong if for example the individual making the decision lives in a locality which is well served by public transport, or where goods and services can be obtained locally, on foot. The presence of this 'external' choice is important in understanding the size of a given market, which will change as the relative 'distances' of the average choice within the system and that of the choices outside change. This sort of effect may be of great importance, for example, in the residential location decisions of an urban population, who may as prices, density and pollution increase within the urban area, switch their preferences to localities outside the 'system' being modelled. Models which ignore this will of course go completely wrong. Similarly, as 'distances' and dissatisfaction' build up in a system, that is with the opportunities available to the individuals, so the relative attraction of 'opting out' may become large.

In other words, the observed behaviour of individuals results from the 'dialogue' between the choices available to them, and their needs and constraints.Adversity can be either a spur to the search for new solution which may lead to an 'instability' and be adopted in the system, or it may lead to a rejection of the system, and to attempts to replace it.

Having digressed somewhat the wider issues underlying observed behaviour and choices, let us now return to the more mundane problems of model building. How can we insert the uncertainty and lack of information which may characterize the pay-offs ? This can be handled quite simply, in the following way. As we have mentioned above it is reasonable to suppose that the probability of making a particular choice is inversely proportional to its 'distance', in the value space of the decider, from the origin. Suppose now however, we wish to consider a situation in which there was no information concerning these distances. If this were so, then clearly, if there are two choices then they have an even change of being selected. On the other hand, if there is absolute certain knowledge that one of the choices is better than the other, then we may suppose that there is a probability of one that it will be adopted. An expression which fulfills these requirements is the following one.

Probability of choice i among all those possible j.

$$P(i) = \frac{A_i}{\sum_j A_j} = \frac{\left(\frac{1}{d_i}\right)^I}{\sum_j \left(\frac{1}{d_j}\right)^I}$$

where I is a measure of the amount of information the individual has to make his decision, A_i is the perceived 'attractivity'.

When $I \longrightarrow 0$ we have equiprobability for all the choices possible, but when $I \longrightarrow \infty$ then we have probability 1 for the choice corresponding to the shortest 'distance'.Ofcourse, we could also take each axis separately and look at the uncertainty in that, since some facts may known precisely, and others extremely vaguely. However, this would make our calculations very much more complicated in the modelling which we shall present here, and so while nothing that that is the correct procedure, we shall not pursue it further.

The main feature of the approach we are attempting to build here, is that for each type of actor, according to his means and his role etc., he will have a different value space. It may be simply a matter of degree, or, it may also be that we have a quite different set of values, probably related to a different 'role' in society.

So far we have been discussing decisional behaviour without taking into account the fact that the system may be evolving.

In a dynamic system the 'pay-offs' which characterize each choice will change in time, as will the choices open to individuals, and this evolution will be predicted by the decision maker according to the 'model' he is either implicitly or explicitly using. It is somewhat disquieting to realize that the models we are going to build will contain the behaviour of actors, which will in turn depend on the models available to them. This is an important point because it may influence the confidence which any forecast may be accorded, and in that case the behaviour of actors may be more dominated by 'satisficing' than 'optimising'.We shall return to this point later because it may be of importance in discussing the use to which modelling should be put; that of predicting the future evolution and of making the 'best' decision, or rather of exploring the 'dangers' and 'uncertainties' of the future and attempting to evaluate acceptable and robust decisions.

In an evolving system then, each actor will attempt to estimate the 'pay-offs' associated with a given choice, not only instantaneously, but also over future times, and according to the importance and weighting which he accords to future times, he will decide which choice he prefers.

Let us now suppose that the probability of making a particular decision i, from $\sum_j$ j, per unit time is proportional to the relative attractivity. From this it is now possible to construct kinetic equations governing the evolution of the numbers of individuals adopting each choice.What is of vital importance however, is that as a given option is adopted so the 'pay-off' (costs, prestige, comfort, etc)will change and so the choice pattern of the population will reflect this, as choices get nearer or farther along the different dimensions of various value systems of the actors.

Consider a homogeneous population x faced with several possible choices.

Then, we may write down that for any particular consumption pattern (i.e. number of clients, S_1, S_2,....S_i consumed) then we will have for the ith choice,

$$\begin{tikzcd} \boxed{i} \arrow[r, bend right, "S_i \frac{A_j}{\sum_i A_j}"'] & \sum_{j\neq i} \boxed{j} \arrow[l, bend right, "S_j \frac{A_i}{\sum_j A_j}"'] \end{tikzcd}$$

Then, if $x = \sum_j S_j$

$$\frac{dS_i}{dt} = a S_i \left(\sum_{j\neq i} S_j \frac{A_i}{\sum_j A_j} - S_i \sum_{j\neq i} \frac{A_j}{\sum_j A_j} \right) \qquad (6)$$

$$\frac{dS_i}{dt} = a S_i \left(x \frac{A_i}{\sum_j A_j} - S_i \right) \qquad (7)$$

and for several populations, x_j each with its own view of the relative attractiveness of the choices, A_{ij}, we have,

$$\frac{dS_i}{dt} = a S_i \left(\sum_j x_j \frac{A_{ij}}{\sum_{i'} A_{i'j}} - S_i \right) \qquad (8)$$

where the attractivity A_{ij} of the i-th choice, viewed by the population type j, is thus some inverse of the 'distance' from the origin of i in the value space of the type j, raised to a power I, as explained above, related to the information available in the system.

In the first example, which we briefly describe here, we have studied the dynamics of a simple market, where products are in competition. We have considered the simplest case, where the 'value' space of the population consists simply of two dimensions : price and quality. The origin of each persons value space is taken to be that his 'ideal' would be the highest quality product imaginable at no cost. In reality, ofcourse, the 'supply' will only offer products where some compromise of price and quality obtains, and customers will be attracted to the various products to different degrees. We have supposed a Gaussian function for the weighting 'accorded' by the different members of the population to the 'price' of a product, which could implicitly or indirectly be related to a Gaussian distribution of income.

Using the equations of type (8) we have studied the dynamic interaction of products, competing for customers, with prices fixed by entrepreneurs by adding a percentage profit to the costs of production, and where we have supposed that these latter increase proportionally with quality.

Now if there were no 'economies of scale', or multiplier effects, no market threshold or psychological effects of fashion, then it is true that an almost infinite number of products could exist each corresponding to the minimum 'distance' d_{ij} in the value space of a particular individual.

However, this is not the case in the real world, where economic and psychological non-linearities abound. Thus, we can show that, depending on the precise sequence of events occuring in the system, that is the moment and size of the launching of each product, as well as the profit margin strategy of each firm, many different stable 'market equilibria' can be attained. For example, the sequence of events may lead, for the same equations of interaction, to either a monopoly, a duopoly or an oligopoly (and in fact many realizations of each), and each of these equilibria is characterized by both qualitatively and quantitatively different flows of goods. In fig.(13) we see the state attained for a particular set of parameter values in equation (13), when all three products start at the same moment with 10 units of production.

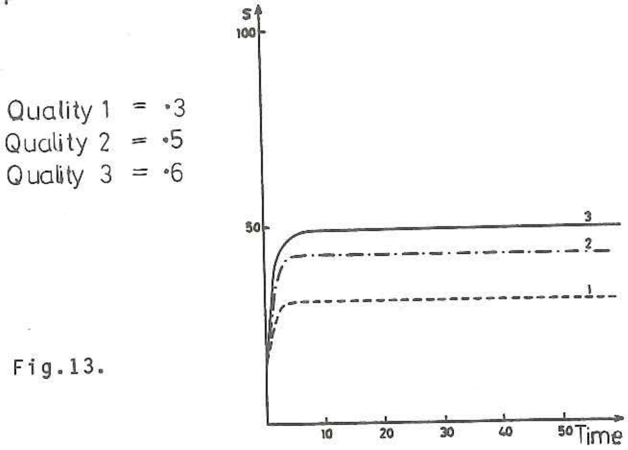


Fig.13.

In fig.(14) however, we see that for the same system there are different possible outcomes which depend simply on the timing of events. For example, if product 1 is launched first then its sales volume moves to the stationary state indicated, assuming that its profit margin remains at 20%. Ofcourse, in the event of monopoly, the firm may modify this profit margin, and this possibility could ofcourse be studied by our model but we shall not concern ourselves with this point here. If product 3 is introduced 10 units of time later, we move towards the duopoly indicated by the levels 1 and 3. If at time t = 20, we attempt to launch the product 2 in the market, we find that an initial size of ten units of production is no longer sufficient to allow its implantation. In fact, it can only establish itself in the market at this moment providing it has an initial production scale of at least 19.5 units. If this is the case then the system evolves towards the stationary state indicated by the levels $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$.

Already, our simple study reveals that the later a firm arrives in a mar ket, the greater the initial investment that is required for it to establish itself. Therefore, whatever 'market equilibrium' we observe for a particular system will depend not only on its precise history, but even on the 'size' of investors that have as yet remained outside this market !

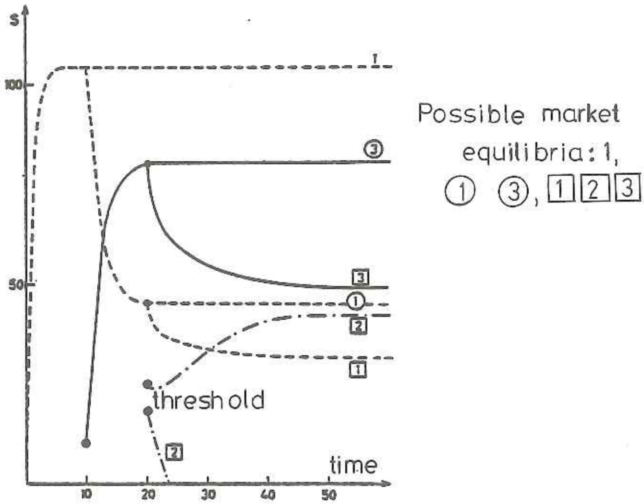


Fig.14.

Similarly, the particular strategies of different entrepreneurs also determines which stationary state is attained by the system, as is shown in figure (20) where we show the possible effects on the market of the decision of a firm to reduce its profit margin. The long term result depends on the type of retaliatory measures adopted by its competitors, and in particular depends on their 'reaction times', for if these are too long then they may fall below the survival threshold.

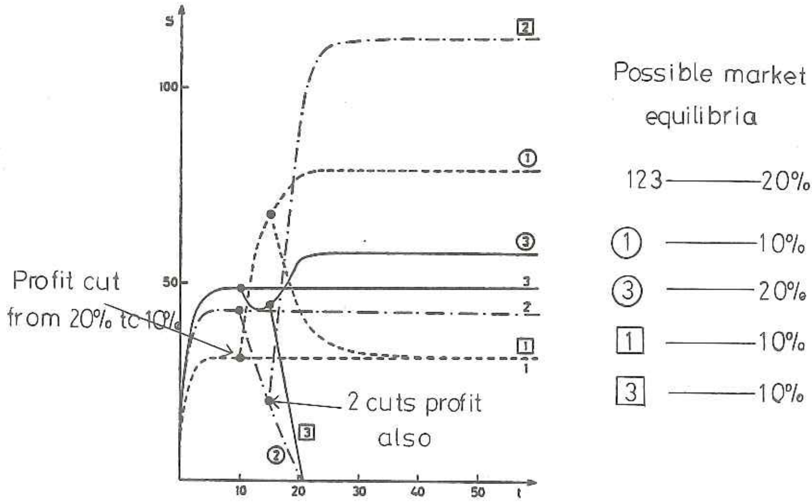


Fig.(15)

We see the irrepressible tendency of an 'unstructed system' of initially equal sized firms to 'structure' or organise itself into a hierarchy of firms, each with different power, to act on the future evolution and the tendency for this structuration to lead to collusion, cooperation and in general an escalation of the 'scale' of coordination involved in the competition. Such a model offers us the basis for an understanding of the origin and evolution of organisational hierarchy in an initially unorganised situation. Ofcourse many other studies have been made using these simple equations, exploring the effects of different market strategies, of cartels, of product specialization etc. but we shall not discuss these further here. (Allen, Frere and Sanglier, 1980).

The most important point of principle is that our analysis shows us that for the same population, having the same 'value system', for the same technology and the same products, the flow of goods in a given market can be both qualitatively and quantitatively different depending only on the 'history' of the system. Thus the fundamental diagram of 'supply' and 'demand' (fig.(16)) is misleading because it can only be constructed in re trospect. It refers to a particular outcome, and the intersection could have been elsewhere. Our model shows us that the 'free market' is not equally open to all agents, since the possibility of successful implantation on an existing market depends on the size of initial investment that can be made.

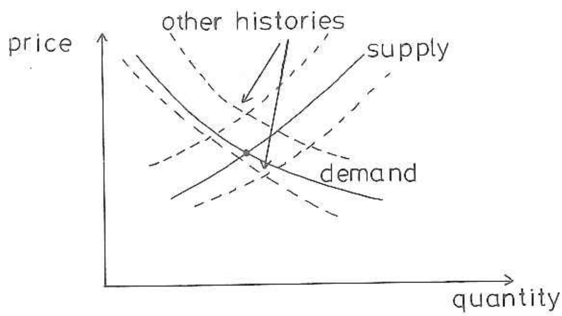


Fig.(16).

Also we see that under some slow change in the parameters of the system (e.g. market size, or economies of scale current interest rates) then a relatively sudden re-organization of the pattern of the consumption could occur when the previous one become unstable. In such changes, relatively small differences of possibly random origin or new initiatives (fluctuations) would prove decisive in the forging of a new structure. Our image of a market system is therefore that of a dynamic 'game' with a varying number of players and stakes, where periods of 'adaptive' jockeying are separated by successive 'crises' or periods of major reorganisation (Day, 1980).

If we consider the long term evolution of our market system then ofcourse the effects of 'innovations' will be of great importance. In general these innovations will occur insome sense, around existing dimensions and structures, causing the system and the 'values' of the population to evolve into new directions, so that societies with different histories will exhibit not only different socio-economic patterns but also in the long run,different 'value systems'. For example, in western society, the automobile was considered simply as an amusing luxury only some 40 or 50 years ago, but the evolution of the system as a whole has led to the fact that it is now viewed as a basic necessity for millions. Similarly, it seems clear from this point of view that the impact of one society on another is a complex and dangerous phenomena, involving a clash of values which the market does not necessarily 'translate' in a neutral manner. Ofcourse, it remains true that the 'market' system does nevertheless involve an exploration of the potential demand among the population for different goods (although it may be imperfect) while this is not necessarily the case for a 'planned economy'. For such a complex system as this there is probably no simple answer to the problem of how the economy should 'best' be run, but then why should there be ?

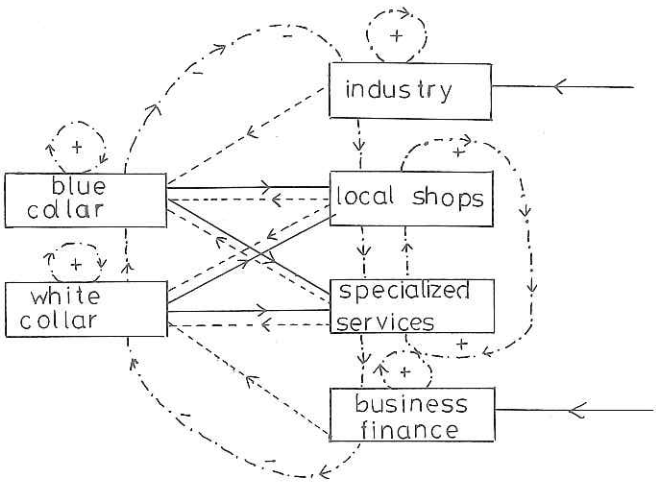
All these points and difficulties are raised clearly when for example, we apply the methods of analysis outlined above to study the evolution of a market in its two spatial dimensions. Recently, urban evolution has been considered from this point of view. We shall not give the details here but simply describe the important points concerning this evolution, which our paradigm of self-organization reveals.

3. THE EVOLUTION OF URBAN STRUCTURES

The urbanization of a region can be studied as economic functions are introduced at different point in the system, and either find a sufficient market and grow, or are eliminated by the competition. The structure that emerges depends on the timing and location of the launching of each economic function, as is therefore merely one of many 'possible' structures which are compatible with the equations representing the economic interactions. It is through the action of elements not explicitly contained in those equations (fluctuations and historical 'accidents') that the choices are in fact made at the various bifurcation points which occur during the evolution of the system. Thus the spatial organization of a region does not result uniquely and necessarily from the 'economic and social laws' enshrined in the equations, but also represents a 'memory' of particular specific deviations from average behaviour. This has been described in detail elsewhere (Allen and Sanglier, 1979), and we shall turn instead to the question of the evolution of urban structure within a city (Allen, Boon and Sanglier, 1980).

In agreement with much previous work, particularly for example the philosophy of a Lowry type model, first we consider the basic sector of employment for the city, and in particular two radically different components of this, the industrial base and the business and financial employment. Next we consider the service employment generated by the population of the city, and by the basic sectors, supposing two levels, a short range set of functions and a long range set. The residents of the city, depending on their type of employment etc. will exhibit a range of socio-economic behaviour, and for this we have supposed two populations corresponding essentially to 'blue' and 'white' collar workers.

The next phase of the modellng is in attempting to construct the interaction mechanisms of these variables, which requires as we have discussed, knowledge of the values and preferences of the different types of actors represented by the variables, and ofcourse how these values conflict and reinforce each other as the system evolves. In fig.(17) we show the basic interaction scheme for six variables whose mutual interaction leads, we suppose, to many of the important features of spatial structure. These variables reflect the decisions, particularly locational decisions, of six basic types of actor.



Demand for Goods and Services

Demand of Labour

Cooperative effects, (economies of scale, common infra-structure, learning, etc.)

Fig.(17) The interaction scheme of our simple City system. -

We then construct our kinetic equations as in the previous section expressing the evolution of each variable, in each locality. As an example, let us write explicitly,

$$\frac{dx_i^k}{dt} = a x_i^k \left(\sum_j J_j^k \cdot \frac{A_{ij}}{\sum_{i'} A_{i'j}} - x_i^k \right) \tag{9}$$

which expresses how the number of residents of socio-economic group k, at the point i, x_{i}, $_{k}$ change in time by the residential decisions of the sum of all those employed in the different possible sectors m, whose jobs are located at j. Thus, A^k_{ij} is the attractivity of residence at i as viewed by someone of socio-economic group k, employed in sector m at the point j.

$$A^{K}_{ij} = \frac{\nu^K \overbrace{(1 + \sigma^K x^K_i)}^{\text{cooperativity}}\, e^{-b^K \overbrace{\Delta_{ij}}^{\text{distance}}}}{\underbrace{\left(\nu^K + \sum_K x^K_i + \sum_L J^L_i\right)}_{\text{crowding}}} \qquad (10)$$

ν^K, σ^K, b^K = characteristic constants

These include the considerations of cost and time in travel to work, the price of land, pollution and noise levels etc., as well as the character of the neighbourhood.

We have written down similar equations for the other actors, which in brief express, for example, the need for industrial employment to be located at a point with good access to the outside, and for a large area of job, as well as some 85% of their workforce being taken to be in the lower socio-economic group. We have also added the fact that the interdependence of many industrial activities leads to a preference for locations adjacent to established industrial locations. This term also covers many subtle effects of the infrastructure that grows around existing situations. The main effects are all noted on the interaction scheme of figure (17).

Here we shall briefly describe some of the simulations that we have made using our simple model. In the first case, we have looked at the evolution of a centre, which initially is only a small town, but throughout the simulation, due to population growth and expanding external demand from the industrial and financial sectors the town grows, spreading and sprawling in space as it does, and also developing an internal structure.

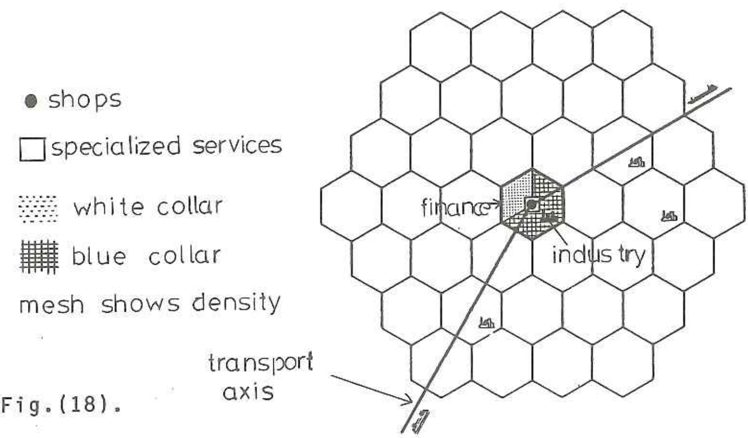


Fig.(18).

The initial condition of the simulation is shown in fig.(18). After 10 units of time, the sitution has evolved to that shown in fig.(19), where already, an internal structure has appeared. Industry, commercial and financial employment are all still located at the centre, but now we observe residential decentralization, particularly on the part of the upper socio-economic group. The centre is very densely occupied and is strongly 'blue collar'.

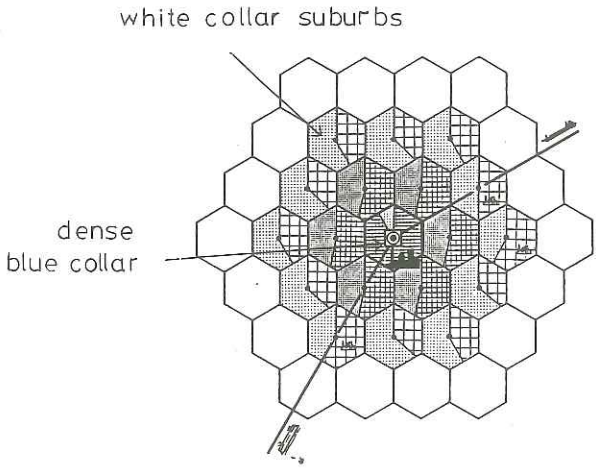


Fig.(19)

As the simulation proceeds however, at around 15 units of time, this urban structure becomes unstable. It is not a question of simply growing of shrinking, what is at issue is the qualitative nature of the structure. For, at this point in time, the very dense occupation of the centre is beginning to make industrial managers think about some new behaviour. For some of them the cost of continuing to operate in the centre, is making them contemplate the abandonment of the infrastructure and mutual dependencies that have grown up with time. At this point, as for a dissipative structure, it is the fluctuations which are going to be vital in deciding how the structure will evolve. At some point there is an initiative, when some brave (if it works out, stupid if it doesn't) individual decides to take his chance and to try to relocate at some point in the periphery. Where exactly, will depend on his particular perceived needs and opportunities. However, what is important is that whereas, before this time such an initiative would have been 'punished' by being less competitive, now, around t = 15, the opposite is true. Once the nucleus is started, and ofcourse its own infrastructure begins to be installed, so almost all the industrial activities decentralize, and establish themselves in this new position in the periphery.

At this point, many different initiatives could succed in carrying the system off to some particular new state of organization. However, those which succeed with the least effort are the industrial nuclei in the periphery, lying along the communication axis.

From this point on, however, the locational decisions of the 'blue collar' workers are particularly affectd by the fact that their value systems are now based on the fact that industrial employment has re-located in the south-western corner of the city. Thus, the spatial distribution of blue collar residents in the city starts to change, having in a sense a new focus. This inturn acts on the locational choices of the white collar workers, who find space easily in the regions of the city less favoured by the blue collars, and whose spatial distribution adjusts accordingly. Changes in the distribution of local service employment also then occur, and the whole structure evolves to the pattern shown in fig.(23) by time t = 40. Here, we see that we have actually displaced the centre of gravity of the urban centre, and have an urban structure which ressembles two overlapping urban centres of different character. In the south west we have predominantly working class, industrial satellite, while, the original city centre is a C.B.D. and important shopping and commercial district, with predominantly white collar suburbs stretching away from it on three sides. In this part of the city, it is the second ring that has attracted the local shopping centres, while in the industrial sattelite, it is the heavily populated, industrial district itself that has become an important shopping centre. From our simulation we can calculate traffic patterns, travel distances and energy costs and we find a complicated behaviour for these.

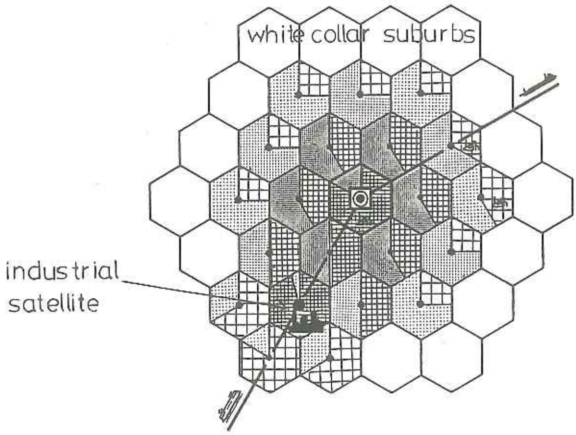


Fig.(19). By time $t=40$, the urban structure has changed qualitatively from that of fig.(9). It has developed a second focus, and has structured functionally. That is, one centre is essentially an industrial stellite, while the traditional centre has become largely a CBD and the important shopping centre. We may also note that in the traditional centre, the retail employment has moved outwards to the second ring, (suburban shopping centres), while in the industrial centre the retail employment is still centralized.

This shows us the dangers involved in global modelling, for on that scale, what we see is an apparently inexplicable change in behaviour, in which the distance travelled per person, and the average energy consumption per person stops rising and even decreases. Only a model which can describe the internal restructuration of the city could have predicted such a change, and linear systems theory, and input-out-put flow models would have to be re-calibrated at this point. In other words, relationships between global variables of complex systems nearly always involve non-linearity and a systems analysis which assumes linearity will only be reasonable in the short term, or in a neighbourhood of the calibration.

CONCLUSIONS

One of the most important points that arises from our discussion and simulations concerns the level of explanation which is aimed at by a model. If we approach a complex system with a desire to model it so that we may understand its evolution and direct our policies more rationally, then clearly, we must first set up what we consider to be the 'structure' of our system. However, if this structure simply reflects the 'structure/function' present in the system at the initial moment, and we then calibrate the 'model' on this initial state, then any 'prediction' that the model makes assumes that the function and structure do not change. This may be quite wrong. Our point of view, derived from the concepts underlying dissipative structure, is that the initial structure/function of the system (pattern of consumption, or where different popultions and jobs are located/traffic flows between them) is itself the result of an evolutionary process which, after a particular history involving both macroscopic and microscopic factors, was established in the system. Because of the existence of multiple solutions, in fact the dynamic equations of the macroscopic variables (the 'model') are ambiguous and could have given rise to 'other' structures if the particular history has been different. Thus, if we admit that the particular initial structure/function of the system with which we start is a 'special case', and that micro factors outside the model led to its establishment, then we must also admit that this will be true of the future evolution. In other words, the future will also have its 'historical accidents' when we look back on it, and although the importance of such events is often widely accepted as concerns the past of a system, modellers have in general not seen the implication for the future.

In order to build models which can cope with such problems, we must therefore look for the underlying interaction processes which can give rise to the many different structure/functions that are observed for different circumstances and histories. The basis of such a search must be human behaviour, outside of explicit statements about space. Thus, the 'structure' of the model should not explicitly contain spatial structure, but this should result from interactions of the humans in the system as they make choices according to their value systems and constraints, choices which arise because of particular initiatives by other actors in the system following the same program but viewed from a diffferent place, and role in society. Part of the choices is indeed that relating to the evolution of the numbers of individuals in each role, and the invention of new roles.

If we look at our interaction diagram for the intra-urban, then we see that this type of approach is indeed initially non-spatial. Thus the interaction cheme could perfectly well exist with identical values of variables at each point of the system. It is, potentially, totally symmetrical. However, because of fluctuations, both in the 'real world' and in the 'mental maps' of individuals, can explore situations which are 'richer' than reduced description of the world which is a model, so this symmetry can be broken, and having been broken can be amplified if some actors perceive an advantage in the new behaviour, and have the 'power' necessary to adopt it. Thus evolution is always characterized by events in which 'abnormal behaviour' becomes 'normal behaviour', when 'informal structure' becomes 'formal'. Small fluctuation are amplified by the advantages perceived by at least some of the actors. Even if such advantages correspond to disadvantages for other actors, then it would depend on the 'power' or 'leverage' of the opposing groups as to whether or not the changes would take place.

Clearly decision is related to perception and by manipulating information one can change the evolution of the system. Both direct advertiring and propaganda as well as social pressure in the form of fads and fashions can create desires and frustrations which may mark the system permanently. Values, it seems, are not the simple, self-evident certainties which we may have believed. Even such 'sure-fire' values a maternal love have recently been shown to be subtle and changing. What must face is that almost all our everyday actions are not the expression of an absolute rationality, but the result of a dynamic dialogue between 'system' and 'values', between 'supply' and 'demand', during which bifurcations occur. Their rationality is simply conferred on them by the society in which they are thought 'normal', where they have evolved, and they can, and will, change. The problem of policy making in a world with changing values is indeed a fundamental one. If we are to ever be able to understand such an evolution, then our models must not simply say : the system is organized like this. They must also examine the question, why is it like this ? The reply will involve necessarily an understanding of the reasons for its stability, and this in turn will allow an appreciation of its potential instability, and of the new dimensions and levels of organization that be created (Jantsch, 1980).

Summarizing the main points made above then, we have examined the behavioural basis of our models and shown how a more systematic inclusion of multiple criteria (both quantitative and qualitative) can be put into the equations. An important general point that arises is that a structural reorganization of say the urban space, leads to a corresponding reorganization of the mental maps and values of the various actors. The symmetry breaking properties of non-linear systems lead to a corresponding expansion of the dimensions of the actors value space. For example, in the case of an initially circular city, the variables and parameters of decisional criteria can all be expressed in terms of the scalar distance from the centre. Once the circular symmetry is broken, however, the value space expands to include all the angle dependent possibilities. Similarly, when all cars where black, the question or value attached to colour was of no importance. Once the symmetry had been broken, however, and cars of other colours appeared, then a new dimension is created in the value space of buyers and finally can become an important factor in sales.

Complexification feeds on itself because it creates new situations and dimensions, which widen the experience of people and create new tastes and qualities, leading to new behaviours and to further complexification and to the creation and destruction of patterns and organizations. Only a much more profound understanding of such self-organization, whose complex nature is merely glimpsed in the above, can help us steer a course in such an unfolding universe of self-discovery.

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