Belgian Journal of Operations Research, Statistics and Computer Science Vol. 36 (2-3) 96

Optimal Location of Undesirable Facilities : a Selective Overview

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Abstract

Traditionally most models in location theory describe situations where nearness to a facility is valued. However many types of facilities have, at least partly, an undesirable effect on the environment, calling for new models with corresponding solution methods, in particular in public locational decision making. Such models have started to be studied by researches in the location field since the early eighties. One important feature is that undesirable effects are usually felt continuously over space, leading to consideration of continuous type distances.

We give here a critical overview of the research in the location of (semi-)undesirable facility location in a continuous space. This field may be considered to still be in its infancy and, although rapidly expanding, offers many opportunities for important, often interdisciplinary research and application.

1 Introduction

1.1 General

Location theory studies problems of locating one or more facilities in some optimal way with respect to several given points with which they will interact. These interactions may represent transports of goods and/or people, physical links like pipelines or cables, communication, attraction of potential customers, but also an undesirable influence like pollution, radiation, etc.

Three broad types of location models are usually distinguished, according to the basic assumptions about the underlying decision space. When a choice has to be made among a (short) finite list of given possible site(s) the location problem is a *discrete* one, and its solution relies on integer and combinatorial optimisation, see e.g. [16] for a recent introduction. In *network location* any point at a node or along an edge of a given graph may be used as a site, with possible additional constraints, and interactions are assumed to be constrained to "movement" along the network; the study of these problems heavily relies on graph theory and one-dimensional optimisation, and often they are amenable to discrete problems by reduction to a finite set of candidate optimal sites, the efficient determination of which often forms the heart of the matter, see e.g. [53] for a recent survey. When the underlying space is determined by continuous variables we obtain *continuous location* problems, the study and solution of which rely on (mainly convex and/or computational) geometry, (mainly convex) analysis, non-linear programming and/or global optimisation, see e.g. [77] for a recent overview.

Traditionally most models describe situations where nearness to a facility is valued, i.e. the interactions are of an attracting kind, and the bulk of all research in the field is concerned with this kind of *attracting location problems*. It is perhaps less well known that in more recent years, in agreement with the current importance and interest in environmental issues, much effort has been spent towards the location of undesirable facilities, where nearness to the facility is detrimental; see the important critical overview of Erkut and Neuman [29]. It seems that continuous location models have an eminent role to play in this field, contrary to its more reduced interest in the location of attracting facilities.

Indeed, at the one hand, (risk of) polluting effects such as airpollution by fumes, radioactivity, radio interference, noise, heat, odour, etc. are felt continuously over geographical space, and there is a direct (although not always simple) relationship between the intensity at one point and its relative spatial position to the generating facility. At the other hand the community is often ready to accept the economical disadvantage of badly accessible sites if these alleviate the detrimental impact of the facility. This implies that, at least in a preliminary study, the set of potential sites may be assumed to consist of some feasible regions. These are exactly the two basic assumptions in any location problem of continuous type.

In this paper we give an overview of the state of the art in optimisation approaches to the location of (semi) undesirable facilities in a continuous space. Special attention is paid to a critical assessment of the assumptions made in the models, to the ease and scope of the proposed solution methods, and to the practical relevance of the offered results. Some suggestions are made towards open problems and possible future research directions. Thus this paper may be viewed at the one hand as a partial update of the excellent seminal survey of Erkut and Neuman [29], here restricted to continuous location, and at the other hand as a complement to our recent broader survey [77] of this latter field which we now feel did not enough justice to undesirable facility location.

1.2 Modelling considerations and classification

Most industrial and other activities have some negative impacts on their environment. These may range from very mild like local disturbance of the electromagnetic field (although perhaps not so friendly to e.g. users of mobile phones), through annoying like noise, up to quite dangerous like toxic fumes. It seems to have become customary to distinguish between *obnoxious*, in the sense of threatening lifestyle through discomfort, and *noxious* facilities, in the sense of dangerous to the health, possibly lethal. This impact may be of ongoing nature, or may be a risk of serious accidents with highly noxious effects associated, such as nuclear power plants (cf. the Three Mile Island and Chernobyl catastrophes), certain chemical plants (cf. Bhopal and Seveso) or petrol and liquid gas treatment and storage plants (cf. the worst-case impact study of a peak-shaving LNG-installation in the Netherlands [90]), to name a few. Such situations are often rather described by the term *hazardous*.

Observe that although the physical effects of the facility may be direct and measurable, indirect effects such as on health are more difficult to measure, often only globally by statistical studies, and several effects may be quite immaterial, and rather subjectively perceived. It is a well documented fact that the undesirability of certain types of facilities, like garbage treatment plants and dumping sites, stems not only from some notorious examples of serious pollution mainly due to bad and illegal management, but mainly from the general feeling of insecurity their nearness engenders, leading to important economic depreciation of property. This exemplifies the rather short term vision most people follow in their personal feelings and reactions about the environment. At the other hand not many hard facts are really known yet about the long-term effects of continuing low-level disturbances, and the consequences on e.g. global health care costs.

When incorporating these kinds of considerations into a mathematical location model the distinctions above usually disappear and only remains the tendency to locate sufficiently far from or as far as possible from the places or regions where the undesirable effects will or could be felt. This is the so-called NIMBY-principle (*Not In*

My BackYard). We therefore prefer the more neutral and quite suggestive term *undesirable facility*, following [29], although the term *repulsive* (or repelling) we used in [77], as opposed to attracting or *push*, used in [26], also adequately describes the way they are modelled.

Erkut and Neuman [29] argue that any decision process for the location of an undesirable facility should consist of two main stages: *site-generation* and *site-selection*. Optimisation seems only suited for the first stage, since the second is usually judged too complex for accurate representation using a single objective model. Any optimisation model is however primarily site-selection oriented, since an (often unique) optimal site is sought. Therefore it seems, at first glance, that no optimisation approach is really suited to this context. We consider, however, that continuous type location models have a strong potential for site-generation purposes through the possibility of constructing level sets of close-to-optimal sites; the main function of the actual optimisation is then no more than to determine the best possible objective value against which to measure the quality of any other site. It is our conviction that in order to be useful in practice, any approach to solving location problems should be able to yield this kind of near-optimality information. As such the continuous location model becomes a first stage in a two (or multiple) stage decision process, by first eliminating all regions for which clearly less detrimental sites may be found. The later stages will then take more aspects into account, as proposed e.g. in [52] and the references therein.

No realistic location model may forget about the myriad constraints imposed upon the possible sites for a facility by geography, policy, politics and jurisdiction. It should therefore allow for the definition of regional constraints of widely different types and adequate solution procedures must be able to handle such a variety.

In any location model of continuous type some notion of distance will be involved. Most undesirable effects are felt continuously over space, so the adequate distance notion should reflect this, which excludes e.g. network shortest path distances (excepting perhaps the directed networks of waterways) or approximations of these, like rectangular or more general ℓ_p-norms (see [57]). What remains are mainly euclidean distance, possibly modified through the impact of winds, currents, natural or artificial barriers and/or nonhomogeneous terrains, and close approximations of these, e.g. by polyhedral norms and gauges with many main directions, yielding problems which may be easier to solve (see [71]). We refer the interested reader to [77] for a more complete overview of all these notions of distance, and further references to the literature.

Typically the intensity of the negative effect should decrease with distance, although notable exceptions to this rule will be described later. This *decay function* is often assumed to be linear, mainly in order to simplify the analysis. As observed in [29] and exemplified in [81], it seems that more realistic models should use convex (or other non-linear) decreasing decay functions, possibly levelling off to zero above a certain "safety" distance. We will argue that discontinuous decay functions may also be of interest.

In most situations the undesirable nature of the facility to be located is only part of the problem and it is simultaneously desirable, otherwise there would be no incentive to actually install it. Most of the currently available models do however disregard the desirable aspects, probably for technical reasons of easy analysis and solution. We will start our survey by taking a look at such *"pure" undesirable facility location problems* in section 2. Within this section we have further classified the different models with respect to the number of facilities to be located, i.e. single facility versus multiple facility models, and with respect to the particular type of objective function used. Some other approaches attempt to model simultaneously the undesirable and the attracting aspects. Section 3 is concerned with such *"semi"-undesirable facility location problems*. In a final section we collected a few models for undesirable facility location which include other aspects than the mere determination of a site, e.g. some routing aspects.

The way the material is presented here expresses only our personal views on the matter and the amount of detail we give is directly related to our impression of applicability for actual location of undesirable facilities. We still tried to cover most currently existing literature up to 1995. No real selection was done other than (sometimes only forthcoming) availability of the full text. Therefore any noncited relevant work probably remained unmentioned only because we were not aware of it, for which we apologise.

2 Pure undesirable facility location

2.1 Single facility location

2.1.1 Minmax

The most popular way of handling undesirability for a single facility is to attempt to put it as far as possible from all sensitive places, by minimising the highest effect on a series of fixed points, which we will call the *affected points*. It may be written in general form as follows :

$$\min_{x \in S} \max_{a \in A} g_a(d(x,a))$$

where

- x denotes the site to be determined for the facility

- $S \subset \mathbf{R}^2$ is the feasible region within which the facility is allowed to be located
- $A \subset \mathbf{R}^2$ is the finite set of affected points at which the undesirable effects of the facility is taken into account
- d is a distance measure, usually the euclidean distance
- g_a is the decay function of the intensity of the undesirable effect with respect to distance. Minimal assumptions are continuity and decrease (or non-increase), a typical example being the gravity type decay functions $g(d) = k/d^2$.

The only paper discussing this *Anti-Rawls-problem* in its full generality is one of the earliest ones [37], showing that there exists at least one optimal site either in the convex hull of A or at a point $s \in S$ *remote* from some $a \in A$, i.e. there are no points of S beyond s along the half-line starting at a and through s. The upper λ-level set of the objective is exactly the union of the balls centred at each $a \in A$, and with respective radius $g_a^{-1}(\lambda)$. Therefore as long as these balls do not cover S any uncovered point x in S improves upon the current value λ. Based on this idea [37] gives an algorithm (there called Black and White) which makes use of the graphical geometric representation of the problem and current level sets, and the pattern recognition power of human vision to discover new candidate points.

When all decay functions are similar, i.e. $g_a(d) = h(w_a d+k_1) + k_2$ (h increasing and w_a,k_1,k_2 constants) the problem simply reduces to the *weighted maxmin problem*

$$\max_{x \in S} \min_{a \in A} w_a d(x,a)$$

which has been independently addressed by a totally similar *interactive graphical* technique in [66], and in [19] when S is an intersection of circles, by an automated version of it using binary search. A different approach, based on complete or partial enumeration of local maxima, was used in [63], [61], [64], [65], [67]. A similar problem, but on the sphere using geodesic distances and without constraints, was addressed in [21], and shown to be equivalent by antipodal transformation to a weighted minmax location problem for which an algorithm is presented.

Except for mathematical elegance and generality, the practical relevance and choice of the weights, or more generally of different decay functions considered in the previous model is not too clear. Therefore one either must take care to check the sensitivity of the outcome with respect to this choice, or just avoid it by considering them all equal. This latter case is discussed in the next paragraph. The former study was prudently undertaken in [30], where the optimal site was traced for weights which were a power of some fixed chosen values, for different powervalues from 1 to 2. Remarkably some sudden jumps were perceived, illustrating the instability of the model with respect to the weights hence the care to be taken when using weights, and also the possible danger in mere optimisation: in such circumstances level set information is of particular interest. A much broader sense study in this vein was started in [9], [10] : the determination of all possible optimal sites for any choice of weights. It turns out that this is the set of *antiefficient* points (excepting the points of A themselves), i.e. those sites for which all distances up to the points of A cannot simultaneously be increased, and a geometrical construction of this set is given for any S being a union of convex polygons.

The case of equal decay functions appears when making the quite realistic assumption that the least possible effect should be felt at any point of A, and that the intensity is totally homogeneous and solely depends on the distance from the emanating site. The location problem then reduces further to the (unweighted) *maxmin problem*, which is equivalent to finding the *largest empty circle* centred in S, in the sense of not containing any point of A in its interior. For euclidean distance d and finite n-point set A this problem is well solved in almost linear time using techniques of computational geometry. The Voronoi diagram, defined by partitioning the plane according to which point of A is closest, is always a planar graph and is constructible in O(nlogn) time. The optimal maxmin site is easily shown to be either at a Voronoi diagram-vertex within S, or at a boundary point of S along some edge of this Voronoi diagram, or still at some extreme point of S; see [83], [80] for this result, and [80], [72] for ample details on Voronoi diagrams. It was shown that all these cases may be constructed and checked in O(nlogn) time when S is the convex hull of A [83], a convex polygon ([15], corrected by [88]) or the intersection of a finite number of circles [82]. [12] offers a simpler direct approach of higher complexity with a program listing in Basic.

Considering only finite sets A of sensitive points is perhaps not very realistic. Therefore it is of interest to be able to also handle infinite sets A. Perhaps the simplest situation of this type appears when A equals the complement of S. The maxmin problem then leads in the general case to *design centering problems* as discussed in [38], where the aim is to include a largest homothetic copy of a given shape (in our case a circle) inside a given region S. For general shapes these are very hard problems for which only some rather inefficient methods of global optimisation are known. For the euclidean distance, and polygonal S, this reduces to the construction of the largest circle fully inside S, as studied in [48], and shown to be related to the skeleton or medial axis concept of computational geometry, mainly stemming from the field of pattern recognition : the medial axis of S is the set of points at equal distance from at least two points of S's boundary. It is always a planar graph, and becomes a tree when S is simple (i.e. without "holes"). Any minmax point will be a node of this graph. Polynomial time construction methods for this graph in the simple case are given in [48],[79] and [54].

An even more applicable model arises when both A and S are unions of polygonal regions, as studied in [41]. The polygons of A are considered to be protected (e.g. urban regions), and thus the facility must lie as far as possible from any point of them. It may again be shown that only a finite number of candidate optimal minmax sites exist, lying on some planar graph closely related to the "area" Voronoi diagram of A (i.e. the diagram obtained by replacing points by the connected components of A), the construction of which was considered in [55] and [8] in relation with collision-free robotic movement.

Similar models with other theoretical assumptions about the underlying space and/or distance measure have been studied. Since we consider these to have only limited application potential in the undesirable facility location field, we mention them here for the sake of (attempted) completeness, but without too many comments. First there are some one-dimensional versions of maxmin problems, such as [58]. The euclidean distance version in three dimensions was studied in [15]. Planar versions but with rectangular (or Manhattan) distance received the attention of several researchers, such as [20], [60], [62] and [2] for the location perspective, and [69], [13], [3] and [73] for the "largest empty rectangle" version of which [34] gives a generalisation to higher dimensions.

2.1.2 Minsum

The minmax objective tries to offer the highest possible protection to all destinations, and may thus be considered to aim at full protection against the undesirable effects of the facility. Another approach consists of minimising the global effect on all destinations. Although quite popular in discrete and network setting, the interest in this kind of objective has been much more reduced in continuous environment. The seminal and often ignored work [37] seems to be the first to formulate and study this type of model under the name *Anti-Weber-problem* :

$$\min_{x \in S} \sum_{a \in A} g_a(d(x,a))$$

with the same assumptions as in previous section. The same localisation result on optimal solutions as in the minmax case holds, for any decreasing decay functions g_a; when the g_a are linear functions only extreme points of the convex hull of S are candidates. Such linear functions (or any concave decreasing decay function) must however be used with care: it may happen that the optimal solution falls exactly at one of the destinations that should be protected. As pointed out in [10] this happens for example in any three point problem and feasible region the triangle formed by them. If however a destination a is feasible and not an isolated point of S, and the corresponding decay function g_a has sufficiently negative slope at 0, then it is shown in [37] that a is not a local minimum. Note that any optimal site will be anti-efficient, and hence the construction of the anti-efficient set in [10] is of direct interest here too.

In order to solve the minsum problem in its full generality a global optimisation technique is necessary, since the objective function is neither quasiconvex nor quasiconcave and usually presents several local minima. Such a method of branch and bound type, called *Big-square-small-square* (BSSS), is described in [37], and is able to handle any S consisting of a union of polygons. It is based on subdividing a rectangular region into four equal subrectangles for branching, and fathoming is done either by nonfeasibility (rectangle fully outside S) or by the calculation of a lower bound. In [76] it was shown how this method may be extended by a second phase in order to generate an approximate near-optimality region.

To our knowledge the only instance of this model which was further studied is the maximisation form with logarithmic decay function. This objective, introduced in [40] without practical motivation as "a prototype version of obnoxious location problem", is shown to be a harmonic function, and as such, may only have optima on the boundary of the feasible region.

Of particular interest in practice, due to the abundance of airborne pollution sources, is modelling the dispersion of pollution carried by the gases expelled from chimneys. This leads to models of a new kind, different from the Anti-Weber problem. Indeed at the one hand this type of polluting effects calls for taking wind into consideration, which is not done when simple distance measures are used, and at the other hand there is no simple decreasing decay function of the polluting effect - very close to the chimney almost no pollution is felt. For a wind of fixed direction and speed, a standard model for pollutants dispersion is the Gaussian plume model, which does not involve distance but rather relative position of facility and point of measurement, as compared to the wind's direction. A first attempt to use this Gaussian plume in a location model is found in [49], [50], where it is shown how the BSSS strategy may be adapted to cope with the complicated structure of the objective. The main modelling difficulty is that in practice the wind direction and speed changes with time. The above mentioned papers considered only the four main wind directions. This approach was extended later [42], [6], [43], [47] to more wind directions. In all this work the versatile BSSS technique was shown to be applicable as general solution strategy, thus allowing for the calculation of near-optimality information.

2.1.3 Other

More general objectives than the minmax and minsum discussed above may be studied. In the context of purely undesirable location problems any decreasing function of the distances might in principle be used. As far as this function remains wellbehaved (e.g. Lipschitz continuous), the BSSS method may be applied with the additional advantage of allowing the generation of near-optimality information [76]. For a slightly more restricted class of functions - decreasing quasiconvex Lipschitz functions of squared euclidean distances (which encompass many of the models discussed so far) - a more direct optimisation technique was developed in [11], based on a generalised version of Kelly's well known cutting plane method, judiciously combined with methods from computational geometry, in particular power diagrams.

When one allows discontinuous decay functions g_a new types of models arise, many of which remain virtually untouched. One typical case arises when some threshold value exists below which the undesirable effects are fully ignored, a feature frequently used by law to obtain simple rulings. Translated into the decay function we obtain a jump to constant zero value for distances above some fixed R. This means ignoring anything beyond distance R from the facility, in other words, to take only into account those affected points lying within distance R from it.

The only such model we are aware of having been studied is obtained when the decay function g_a remains constant at some weight w_a (e.g. the population at a), and cuts off to 0 above distance R. Effectively this means finding the circle of given radius R covering the least total or maximum weight of destinations yielding minsum (resp. minmax) covering models. Such *mincovering problems* are studied in [25] with a given circle as feasible region S (the same paper also considers the rectangular balls case, of less interest to us here). The minsum approach is to draw all circles with radius R centred at the destinations. Their intersections with S partition S into regions of constant objective value, which may be scanned and evaluated in $O(n^2 \log n)$ steps. The minmax case is solved in $O(n \log^2 n)$ steps by a binary search over the weights, checking feasibility by a largest empty circle routine applied to those points with weight within the currently checked one. Without binary search, but repetition for each of the n possible weights, the same technique yields the largest possible R for each maximal covered weight, thus solving the minmax case simultaneously for all r in $O(n^2 \log n)$ time.

In [78] it is shown how the minsum problem may also be solved simultaneously and for all values of R in a total of $O(n^3 \log n)$ steps. In fact the method generates all Pareto optimal solutions for the bicriterion problem of maximising R while minimising the covered weight, for which only $O(n^3)$ candidates are shown to exist. The interest is to be able to measure the trade-off between technology (choice of R) and impact on environment (covered weight). This allows for a rational choice of technology, thus of R, and, once made, it is easy to draw geometrically the corresponding level sets, yielding a complete description of the shape of the mincovering objective. The approach also applies to the realistic case where the feasible region S is a network, e.g. a road-network.

A mincovering model for fixed distance R restricted to a planar network was also considered in [84] but now the affected set A is the full network, i.e. the affected weight is due to both (pieces of) edges and nodes.

2.2 Multiple facility location

As usual in the continuous location field one finds much less work on multiple facility location models, very probably due to the fact that these are usually rather untractable, certainly as compared to their single facility versions. In view of the fact that single undesirable facility models are already quite hard to handle, this is even more the case for multiple undesirable facility location. The earlier survey [29] mentions only one, which we classify as semi-undesirable (see next section), and we were able to find only a few new references.

The *p-dispersion* problem asks for locating p facilities within some region and as far as possible from each other, in the sense of maximising the minimal distance between any two of them. This may be seen to be equivalent to the *p-circle packing problem* which asks to pack, without overlaps, p circles of maximal equal radius into a given region (see [17]). A nice introduction to this long-standing and popular problem in recreational mathematics will be found in [33], focusing on circular and square regions. Some additional results are given in [17]. For the equilateral triangle region case [35] contains a wealth of recent results and gives further references. For our purposes the most important information is probably found in [17], where a formulation is given as a non-linear programming problem together with the message that the direct and repetitive use of standard non-linear-programming codes may be considered as a good heuristic, while [35] reports on a special purpose "billiards" simulation algorithm. A polynomial time heuristic, yielding a solution with radii guaranteed to be within half of the best possible radius and applying to any metric space is discussed in [86]. Although probably less relevant here we wish to mention also the well known three-dimensional *sphere packing* problem, for which a conjecture of Kepler dating from 1611 still seems to hold, although allegedly solved by [39], where one will find ample further references. The rectangular distance version of this problem is studied in [89] but remains of rather theoretical interest.

There seems to be only one paper, [7] which looks at the problem of locating several facilities in a continuous space so as to maximise both their interdistances and the distances up to a given set of affected points. It concerns however rectangular distances, on which we decided not too spend too much attention.

2.3 Suggestions

Many single undesirable facility location problems remain unstudied and offer numerous opportunities for research in view of their high practical interest. We might suggest following features as worthy of further study.

Most models only consider affected points, although this assumption is certainly not valid in practice. Continuously distributed affected points, e.g. urban settlements, should be incorporated. The one-dimensional study [74] using a negative exponential decay function may be seen as a vague first step in this direction.

There is a great opportunity for interdisciplinary work in the specification of good objectives, reflecting not only the way the direct physical effects spread, but also the way the indirect effects are felt, and thus how they should be treated analytically.

Apart from the Gaussian plume model for airborne pollution, no other attempts seem to have been made to incorporate the spatial heterogeneity inherent in pollution dispersion. At the one hand the asymmetry due to winds still calls for more study, while no good way seems as yet to have been devised to integrate the changes over time as expressed by the windrose. At the other hand waterborne pollution spreads in very different, typically unidirectional ways, and at several speeds according to the type of surrounding (river, groundwater, seepage, etc.). It seems to us that the recently studied distance notions in heterogeneous terrains (see e.g. [68]) hold promise to be useful in this context.

All models we have mentioned seem to assume that locating undesirable facilities is started from scratch. This is of course untrue in most situations. One might say that undesirable facilities already abound, and their presence should be taken into account by the model, otherwise the same sites will crop up as best over and over again with as a direct consequence the ensuring tendency to become garbage "mountains". Here some notion of equity (see also section 3) must be used in the sense that it is unfair to go on locating undesirable facilities close to places that already carry a heavy burden from their environment.

Clearly many opportunities exist in the study of multiple undesirable facility location models. The main difficulty is that it is not very useful to define models without corresponding solution procedure. Exact optimisation methods seem totally out of question due to the highly non-standard objective function shapes one obtains - although no one as yet seems to have tried out existing global optimisation techniques in this setting. One may wonder whether the BSSS method - which is so powerful in the single facility case - could not be applied to multifacility problems of low order. As for heuristics, many of the now popular metaheuristics - neural networks, simulated annealing, Tabu search, genetic algorithms - could perhaps favourably be put into practice here, possibly combined with some of the existing single-facility techniques as local search subroutine.

3 Semi undesirable facility location

We will call a location model of the semi-undesirable type if in some way it includes both undesirability and attracting aspects. This may be expressed in different ways, either in terms of constraints or within the objective. Thus three single objective cases are distinguished, treated in the three following sections. Another, probably more adequate approach would consist in considering undesirability and attraction as two separate criteria. This area of bi-objective (or multi-objective) seems, however, still unexplored.

3.1 Undesirability as a constraint

As a first case we consider classical location problems of the attracting type where undesirability is included in terms of a constraint. Such a constraint should take the form of either a forbidden region, or a minimum distance up to some protected points. These situations were adequately called *restricted* location problems, and are quite fully described, analysed and solved in [36], where one will find more references. Some extensions to the location of two facilities are found in [70].

Other interesting papers in this context, [51], [4] and [1], include also barriers to travel, i.e. regions through which transportation of (in our case hazardous) goods is not allowed, and as such may also be viewed as location-routing.

3.2 Attraction as a constraint

In this category fall the papers mentioned in section 2 with a feasible region S defined as an intersection of balls. Indeed, the objective is one of undesirability, and the constraints may be viewed as "maximum distance constraints" in the sense of not letting the facility be too far away. The two most notable papers in this connection are [19] and [82] and were discussed before.

As announced higher, one multiple facility study should be mentioned here. In [22] the problem is tackled of locating p facilities so as to maximise the weighted distance of any destination to the closest facility, while keeping these distances within given maximal ranges. An algorithm $O(n^3)$ is provided in one-dimension. This was strongly improved in [85] where an O(nlogn) method is described. An extension to the more applicable two-dimensional case is lacking.

3.3 All in one objectives

In this final category we take a look at several kinds of objective functions which may be viewed as including both undesirability and attractiveness. Mathematically speaking this means that the objective to be minimised, viewed as a function which transforms the set of all distances up to the destinations into one global value, is partly increasing (attracting part) and partly decreasing (undesirable part) or, to use the terms introduced in [26], includes both pull and push effects. Although easily stated, it is not always very clear how to apply such models in practice. Since both the attraction (usually some kind of global transport cost) and the undesirability (not always well measurable) are mixed in the objective, this means in practice that these two aspects are considered to be at least comparable, if not substitutable. Before applying the model this assumption should first clearly be validated.

Probably the simplest and best known problem of this type is the *Weber problem with attraction and repulsion*, including both positive and negative weights, i.e. a minsum problem with linear cost and decay functions. The first generally available paper is [87] in which the three destination point case is solved and a statistical analysis is given of cases with more points, emphasising when such a point is optimal. In [24] this problem is studied without locational constraints, and it is shown that the behaviour of the model strongly depends on the sum of all weights. If it is positive, then the optimal solution remains within finite bounds, but moves off to infinity when negative; when zero, both behaviours may occur. Exact solution methods are derived for rectangular and for squared euclidean distances (for which previous results do not hold : there is always a simple optimal solution). For the, in our context, most interesting euclidean case the authors derive exact conditions for a destination point to be the optimal site - for more generally valid sufficient conditions of "majority" type see [75] - and construct a circular region containing the optimal site when total weight is positive. A Weiszfeld-like iterative method is given, supposedly convergent to a local minimum. The first efficient exact solution method, valid also for constrained problems where S is any finite union of convex polygons, is described in [14]. It is based on special techniques of d.-c. programming, i.e. the minimisation of a difference of two convex functions.

Several other problems may be classified in this part, but it is sometimes unclear whether they really apply to undesirable facility location, apart from a tendency to often yield optimality at infinity when no constraints are present. We just mention them here for sake of completeness. In [18] the objective is to minimise the range of distances, in other words the difference between maximum and minimum euclidean distance. A similar objective but with weights on both parts is studied in [59] for rectangular distances. These types of objective functions may be viewed as inequality measures, as adequately explained in [28], where other proposals may be found. Unfortunately, for our purposes however, this paper looks at inequality in the framework of attracting facilities, and it would be worthwhile to take a closer look at the particularities of inequality with respect to undesirable facilities, since it is in this context that the word "equity" appears quite often, be it usually in a very loose way.

4 Location-routing and route location

Perhaps one of the most important remarks in [29] is that "the location of an undesirable facility is almost always connected with the establishment of an undesirable network (high voltage power grids, LNG pipelines,...)", or the use of an existing network for the transport of hazardous material. Therefore the choice of a site should often also take these routing aspects into account. One may say that this field is still widely open. Several surveys of material relevant to these questions of *hazardous materials logistics* have recently appeared ([56], [5], [31], [91]), and we refer the interested reader to these.

We also want to mention here the interesting study [45] of how to locate an air polluting plant along a network so as to have minimal global polluting impact on a number of points. What is remarkable here is the fact that airborne pollution is modelled according to the Gaussian plume model, so the objective is of complicated continuous type, while the feasible region consists of the edges of an existing network. A slightly connected and somewhat complementary study [27] is concerned with locating a facility outside a network (in a discrete setting), but taking into account both transport costs and construction cost of a new link connecting the facility site to the network.

The design of a route between two points which avoids as much as possible some given sensitive points has been studied in [23] with a maximin objective, and in [44] with a minisum objective. It should be mentioned that this type of problem is quite popular in robotics for automatic obstacle avoidance, and has been intensively studied by way of computational geometry, see e.g. [72]. The choice of routes on a network so as to minimize the possible pollution by air on several population centres due to an accident along the route is studied in [46]. Finally [32] propose a general framework to assess the risk involved when transporting hazardous material, which remains as yet unexplored both for route design and location modelling.

References

1. ANEJA,Y.P., PARLAR,M., (1994), Algorithms for Weber facility location in the presence of forbidden regions and/or barriers to travel, Transportation Science, 28, 70-76.
2. APPA,G., GIANNIKOS,I., (1994), Is linear programming necessary for single facility location with maximin of rectilinear distance ?, Journal of the Operational Research Society, 45, 97-107.
3. ATTALAH,M.J., FREDERICKSON,G.N., (1986), A note on finding a maximum empty rectangle, Discrete Applied Mathematics, 13, 87-91.
4. BATTA,R., GHOSE,A., PALEKAR,U., (1989), Locating facilities on the manhattan metric with arbitrarily shaped barriers and convex forbidden regions, Transportation Science, 23, 26-36.
5. BOFFEY,T.B., KARKAZIS,J., (1993), Models and methods for location and routing decisions relating to hazardous materials, Studies in Locational Analysis, 5, (Isolde VI survey papers), 149-166.
6. BOFFEY,T.B., KARKAZIS,J., MALEVRIS,N., (1992), Location of facilities producing airborne pollution, Journal of the Operational Research Society, 43, 313-320.
7. BRIMBERG,J., MEHREZ,A., (1994), Multi-facility location using a maximin criterion and rectangular distances, Management Science, 35, 645-6742, 11-19.
8. CANNY,J., DONALD,B., (1988), Simplified Voronoi diagrams, Discrete and Computational Geometry, 3, 219-236.
9. CARRIZOSA,E., (1992), *Problemas de localizacion multiobjetivo* (spanish), Ph.D. Thesis, Facultad de Matematicas, Universidad de Sevilla, Spain.
10. CARRIZOSA,E., MUNOZ,M., PUERTO,J., (1995), On the set of optimal points to the weighted maxmin problem, Studies in Locational Analysis, 7,21-33.
11. CARRIZOSA,E., PLASTRIA,F., (1995), Locating an obnoxious facility by generalized cutting planes, Working Paper, ESTIO, Facultad de Matematicas, Universidad de Sevilla, Spain.
12. CASINI,E., PAPINI,P.L., (1986), Un algoritmo per determinare nel piano un certo cerchio massimo (an algorithm to determine a certain maximal circle in the plane) -italian, Ed. CIFARELLI,"*Scritti in onore di F.Brambilla*" , Bocconi Comunicazione, Milano, Italy, 145-152.
13. CHAZELLE,B., DRYSDALE,R.L., LEE,D.T., (1986), Computing the largest empty rectangle, SIAM Journal on Computing, 15, 300-315.
14. CHEN,P.C., HANSEN,P., JAUMARD,B., TUY,H., (1992), Weber's problem with attraction and repulsion, Journal of Regional Science, 32, 467-486.
15. DASARATHY,B., WHITE,L.J., (1980), A maximin location problem, Operations Research, 28, 1385-1401.
16. DASKIN,M., (1995), *Network and Discrete Location: Models, Algorithms and applications*, Wiley.
17. DREZNER,Z., ERKUT,E., (1995), Solving the continuous p-dispersion problem using non-linear programming, Journal of the Operational Research Society, 46, 516-520.

18. DREZNER,Z., THISSE,J.F., WESOLOWSKY,G.O., (1986), The minimax-min location problem, Journal of Regional Science, 26, 87-101.

19. DREZNER,Z., WESOLOWSKY,G.O., (1980), A maximin location problem with maximum distance constraints, American Institute of Industrial Engineers Transactions, 12, 249-252.

20. DREZNER,Z., WESOLOWSKY,G.O., (1983), The location of an obnoxious facility with rectangular distances, Journal of Regional Science, , 241-248.

21. DREZNER,Z., WESOLOWSKY,G.O., (1983), Minimax and maximin facility location on a sphere, Naval Research Logistics Quarterly, 30, 305-312.

22. DREZNER,Z., WESOLOWSKY,G.O., (1985), Location of multiple obnoxious facilities, Transportation Science, 19, 193-202.

23. DREZNER,Z., WESOLOWSKY,G.O., (1989), Location of an obnoxious route, Journal of the Operational Research Society, 40, 1011-1018.

24. DREZNER,Z., WESOLOWSKY,G.O., (1991), The Weber problem on the plane with some negative weights, INFOR, 29, 87-99.

25. DREZNER,Z., WESOLOWSKY,G.O., (1994), Finding the circle or rectangle containing the minimum weight of points, Location Science, 2, 83-90.

26. EISELT,H.A., LAPORTE,G., (1995), Objectives in location problems, in Facility location: a survey of applications and methods, Z.DREZNER (Ed.), Springer, 151-180.

27. ERKUT,E., (1992), A mixed-norm location problem for an undesirable facility, Research Report 92-1, Dept. of Finance and Management Science, Faculty of Business, University of Alberta, Canada.

28. ERKUT,E., (1993), Inequality measures for location problems, Location Science, 1, 199-217.

29. ERKUT,E., NEUMAN,S., (1989), Analytical models for locating undesirable facilities, European Journal of Operational Research, 40, 275-291.

30. ERKUT,E., ÖNCÜ,T.S., (1991), A parametric 1-maximin location problem, Journal of the Operational Research Society, 42, 49-55.

31. ERKUT,E., VERTER,V., (1995a), Hazardous materials logistics : a review, in Drezner,Z. (ed.), *Facility location : a survey of applications and methods*, Springer, p.467-506.

32. ERKUT,E; VERTER,V., (1995b), A framework for hazardous materials transport risk assessment, Risk Analysis, 15, 589-601.

33. GARDNER,M., (1992), *Fractal music,Hypercards and more ...*, W.H.Freeman, New York.

34. GOETZE,B., GORDEJEV,E.N., NEHRLICH,W., KOCHETKOV,D.W., (1990), Maximal empty boxes in higher dimensional spaces, Journal of Information Processing and Cybernetics (form. Elektron.Inf.ver.Kyb.), 26, 537-545.

35. GRAHAM, R.L., LUBACHEVSKY,B.D., (1995), Dense packings of equal disks in an equilateral triangle: from 22 to 34 and beyond, The electronic journal of combinatorics, 2, 1-39.

36. HAMACHER,H.W., NICKEL,S., (1995), Restricted planar location problems and applications, Naval Research Logistics, 42, 967-992.

37. HANSEN,P., PEETERS,D., THISSE,J.F., (1981), On the location of an obnoxious facility, Sistemi Urbani, 3, 299-317.

38. HORST,R., TUY,H., (1990), *Global Optimization, deterministic approaches*, Springer.

39. HSIANG,W.Y., (1993), On the sphere packing problem and the proof of Kepler's conjecture, International Journal of Mathematics, 4, 739-831.

40. KAISER,M.L., MORIN,T.L., (1993), Center points, equilibrium positions, and the obnoxious location problem, Journal of Regional Science, 33, 237-249.

41. KARKAZIS,J., (1988), The general unweighted problem of locating obnoxious facilities on the plane, Belgian Journal of Operations Research Statistics and Computer Science, 28, 3-49.

42. KARKAZIS,J., (1990), The discrete n-wind approach for solving the problem of locating facilities causing airborne pollution, In ORBAN,F. and RASSON,J.P., Proceedings of the Fifth Meeting of the EURO Working Group on Locational Analysis, Sept.3-6, 1990, Namur, Belgium.

43. KARKAZIS,J., (1991), The problem of locating facilities causing airborne pollution revisited, OR Spektrum, 13, 159-166.

44. KARKAZIS,J., (1992), Routing of protected materials/commodities on the plane : determining routes in an unfriendly environment, Proceedings of the 6th meeting of the EURO Working Group on Locational Analysis,J.A.MORENO-PEREZ (Ed.),Universidad de la Laguna (Spain), Ser.Inf.34.

45. KARKAZIS,J., BOFFEY,T.B., (1995a), Modelling pollution spread, Studies in Locational Analysis, 7, 91-104.

46. KARKAZIS,J., BOFFEY,T.B., (1995b), Optimal location of routes for vehicles transporting hazardous materials, European Journal of Operational Research, 86, 201-215.

47. KARKAZIS,J., BOFFEY,T.B., MALEVRIS,N., (1992), Location of facilities producing airborne pollution, Journal of the Operational Research Society, 43, 313-320.

48. KARKAZIS,J., KARAGIORGIS,P., (1986), A method to locate the maximum circle(s) inscribed in a polygon, Belgian Journal of Operations Research Statistics and Computer Science, 26, 3-36.

49. KARKAZIS,J., PAPADIMITRIOU,C., (1988), Optimal location of facilities causing atmospheric pollution, PELEGRIN,B.(ed.),Proceedings of III working group in locational analysis,Sevilla (WP ORLOF1, Dept Bus.Adm. Univ. of the Aegean, 82100 Chios, Greece).

50. KARKAZIS,J., PAPADIMITRIOU,C., (1992), A branch-and-bound algorithm for the location of facilities causing atmospheric pollution, European Journal of Operational Research, 58, 363-373.

51. KATZ,I.N., COOPER,L., (1981), Facility location in the presence of forbidden regions I : Formulation and the case of euclidean distance with one forbidden circle, European Journal of Operational Research, 6, 166-173.

52. KLEINDORFER,P.R., KUNREUTHER,H.C., (1994), Siting of hazardous facilities, in *Operations research and the public sector*, Handbooks in OR & MS, Vol6, S.M. Pollock et al (Eds), Elsevier, 403-440.

53. LABBE,M., PEETERS,D., THISSE,J.-F., (1995), Location on networks, in M.Ball,T.Magnanti, C.Monma, G.Nemhauser (Eds.), *Handbooks of Operations Research and Management Science : Network Routing*, North-Holland, 551-624.

54. LEE,D.T.,(1982), Medial axis transformation of a planar shape, IEEE Transactions on Pattern Analysis and Machine Intelligence, 4, 363-369.

55. LEVEN,D., SHARIR,M., (1987), Planning a purely translational motion for a convex object in two-dimensional space using generalized Voronoi diagrams, Discrete and Computational Geometry,2, 9-31.

56. LIST,G.F., MIRCHANDANI,P.B., TURNQUIST,M.A., ZOGRAFOS,K.G., (1991), Modeling and analysis for hazardous materials transportation : risk analysis, routing/scheduling and facility location, Transportation Science, 25, 100-114.

57. LOVE,R.F., MORRIS,J.G., WESOLOWSKY,G.O., (1988), *Facilities location, models & methods*, North-Holland.

58. MEHREZ,A., SINUANY-STERN,Z., STULMAN,A., (1983), The one-dimensional single facility maximin distance location problem, Journal of Regional Science, 23, 233-239.

59. MEHREZ,A., SINUANY-STERN,Z., STULMAN,A., (1985), A single facility location problem with a weighted maximin-minimax rectilinear distance, Computers and Operations Research, 12, 51-60.

60. MEHREZ,A., SINUANY-STERN,Z., STULMAN,A., (1986), An enhancement of the Drezner-Wesolowsky algorithm for single-facility location with maximum of rectilinear distance, Journal of the Operational Research Society, 37, 971-977.

61. MELACHRINOUDIS,E., (1985), Determining an optimal location for an undesirable facility in a workroom environment, Applications of Mathematical Models, 9, 365-369.

62. MELACHRINOUDIS,E., (1988), An efficient computational procedure for the rectilinear MAXIMIN location problem, Transportation Science, 22, 217-223.

63. MELACHRINOUDIS,E., CULLINANE,T.P., (1982), A maximin approach to the location of an undesirable facility in a nonconvex region, Modelling and Simulation, Vol 13,2, Proceedings of the thirteenth Annual Pittsburgh Conference, Pittsburgh, USA, April.

64. MELACHRINOUDIS,E., CULLINANE,T.P., (1985), Locating an undesirable facility within a geographical region using the maximin criterion, Journal of Regional Science, 25, 115-127.

65. MELACHRINOUDIS,E., CULLINANE,T.P., (1985), A heuristic approach to the single facility maximin location problem, International Journal Of Production Research, 23, 523-532.

66. MELACHRINOUDIS,E., CULLINANE,T.P., (1986), Locating an undesirable facility with a minimax criterion, European Journal of Operational Research, 24, 239-246.

67. MELACHRINOUDIS,E., MACGREGOR SMITH,J., (1995), An $O(mn^2)$ algorithm for the maximin problem in E^2, Operations Research Letters, 18, 25-30.

68. MITCHELL,J.S.B., PAPADIMITRIOU,C.H., (1991), The weighted region problem: finding shortest paths through a weighted planar division, Journal of the ACM, 38, 18-73.

69. NAAMAD,A., LEE,D.T., HSU,W.L., (1984), On the maximum empty rectangle problem, Discrete Applied Mathematics, 8, 267-277.

70. NICKEL,S., (1994), Restricted and bicriterial 2-facility location problems, in *Operations Research '93*, Physica verlag- Springer, 366-369.

71. NICKEL,S., (1995), *Discretization of planar location problems*, Shaker, Aachen.

72. OKABE,A., BOOTS, B., SUGIHARA,K., (1992), *Spatial Tesselations : Concepts and applications of Voronoi diagrams*, Wiley, Chichester.

73. ORLOWSKI,M., (1990), A new algorithm for the largest empty rectangle problem, Algorithmica, 5, 65-73.

74. PAROUSH,J., TAPIERO,C.S., (1976), Optimal location of a polluting plant on a line under uncertainty, Journal of Regional Science, 16, 365-374.

75. PLASTRIA,F., (1991), The effect of majority in Fermat-Weber problems with attraction and repulsion in pseudometric spaces, Yugoslav Journal of Operations Research, 1, 141-146.

76. PLASTRIA,F., (1992), GBSSS, The generalized big square small square method for planar single facility location, European Journal of Operational Research, 62, 163-174.

77. PLASTRIA,F., (1995), Continuous location problems, in *Facility location. A survey of applications and methods*, (Z.Drezner, ed.), Springer, 225-262.

78. PLASTRIA,F., CARRIZOSA,E., (1995), Undesirable facility location in the euclidean plane with minimal covering objectives, BEIF Working Paper, Vrije Universiteit Brussel.

79. PREPARATA,F.P. (1977), The medial axis of a simple polygon, In Proceedings of the sixth Symposium on Mathematical Foundation of Computer Science, Lecture Notes in Computer Science, 53, Springer, 443-450.

80. PREPARATA,F.P., SHAMOS,M.I., (1985), *Computational geometry - an introduction*, Springer Verlag.

81. RAHMAN,M., KUBY,M., (1995), A multiobjective model for locating solid waste transfer facilities using an empirical opposition function, INFOR, 33, 1, 34-49.

82. RANGAN,C.P., GAVINDAN,R., (1992), An o(nlogn) algorithm for a maxmin location problem, Discrete Applied Mathematics, 36, 203-207.

83. SHAMOS,M.I., HOEY,D., (1975), Closest-point problems, Sixteenth annual IEEE Symposium on Foundations of Computer Science, 151-162.

84. SUNG,G.S., JOO,C.M., (1994), Locating an obnoxious facility on a euclidean network to minimize neighbourhood damage, Networks, 24, 1-9.

85. TAMIR,A., (1988), Improved complexity bounds for center location problems on networks by using dynamic data structures, SIAM Journal of Discrete Mathematics, 1, 377-396.

86. TAMIR,A., (1991), Obnoxious facility location on graphs, SIAM Journal of Discrete Mathematics, 4, 550-567.

87. TELLIER,L.N., POLANSKI,B., (1989), The Weber problem : frequency and different solution types and extension to repulsive forces and dynamic processes, Journal of Regional Science, 29, 387-405.

88. TOUSSAINT,G.T., (1983), Computing largest empty circles with location constraints, International Journal of Computer and Information Sciences, 12, 347-358.

89. TRIETSCH,D., (1993), Optimal multifacility defensive location on planes with rectilinear distances, Networks, 23, 517-523.

90. VAN DIJK,J.J., ROSING,K.E., (1989), Vloeibaar aardgas op de Maasvlakte - (Liquid natural gas on the Maasplain, in dutch), ESB - Economische en Sociale Berichten (The Netherlands), 6-9, 877-879.

91. VERTER,V., ERKUT,E. (1994), Hazardous materials logistics : an annotated bibliography, in C.Carraro, A.Haurie (eds.) *Operations Research and Environmental Management*, Kluwer, Academic Publishers, 221-267.