Availability of three–machine two–buffer systems

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Abstract

The flow–shop studied consists of three machines arranged in series through which material must pass. Each machine is subject to failure and, to improve the overall availability of the system, buffer storages are located between each pair of subsequent machines. Analytical approaches address to the formulation of a system of balance equations describing the transition of the system state. The number of states is quite high. An approximation is formulated in case the time to failure and time to repair is exponentially distributed. A simulation is built for this system to evaluate the validity of the approximation.

Keywords : transfer lines, interstage buffers, production availability, unreliable machines

1. INTRODUCTION

The problem of designing and improving flow line production systems has received a great deal of attention in the literature. These production systems consist of a number of stages (arranged in series) at which operations are performed on a workpiece. Operations at the stages are performed by machines or by equipment which are subject to failure. Failures at any stage result in the failure of the entire production system and consequently the overall production rate is affected.

In order to improve the production rate, basically there are two approaches. One is the utilization of stand-by machines, the other is to allocate buffer storages between the stages. The decision how to allocate buffers to a production line is of practical importance to the industry, especially to those with assembly machines, canning, and packaging lines.

In this note, the problem of determining the availability of a three-machine two-buffer system is addressed, using an approach which is an extension of the technique by MALATHRONAS et al. [1983]. They study the problem of the availability of a two-machine system with one intermediate buffer. A simple approach to determine the optimal sizes of the two buffers in the three-machine system is also presented assuming rather general storage conditions.

In the literature on multi-stage lines with finite intermediate buffers, DE KOSTER [1989] distinguishes four classes of models. A *first* class deals with systems in which the service times are random variables and the products are discrete. The machines are not susceptible to failure. An example of such a system is studied in KRAEMER and LOVE [1970].

A *second* class assumes deterministic processing times, but machines are unreliable and fail from time to time. Products are discrete. Examples can be found in OKAMURA & YAMASHINA [1977], in SHESKIN [1976] and in SUTALAKSANA & VAN WASSENHOVE [1981,1982]. Mostly life and repair times are geometrically distributed.

A *third* class deals with continuous flow models. Machine speeds are deterministic but machines may fail. This is the class of models this paper will deal with. Some examples of these models can be found in BUZACOTT & HANIFIN [1978], MURPHY [1975, 1978], FOX & ZERBE [1974], WIJNGAARD [1979], MALATHRONAS et al. [1983], COILLARD & PROTH [1984], YERALAN et al. [1986] and MITRA [1988].

A *fourth* class deals with models with discrete products, failures of machines and random processing times. This can be found e.g. in BUZACOTT [1972] and in GERSHWIN & BERMAN [1981].

2. NOTATIONS AND ASSUMPTIONS

The performance of a production system is measured by its *availability* defined as the ratio of uptime by total time. In some papers other terminology is used: e.g. BUZACOTT & HANIFIN [1978] use *efficiency* defined as the ratio

$$A = \lim_{t\to\infty} q(t)/Q(t)$$

where

$q(t)$ = actual production in time t
$Q(t)$ = what production in time t would have been with no stoppages,

or the *throughput* [DE KOSTER, 1989, p. 31] defined as

$$\lim_{t\to\infty} \frac{\int_0^t q(\tau)\, d\tau}{t}$$

where $q(\tau)$ stands for the line output at time τ.

The production lines are assumed to produce at a constant rate and to have unreliable machines.

The availability of a single machine can be expressed as:

$$A(t) = \frac{MTTF}{MTTF + MTTR} = \frac{up\ time}{up\ time + down\ time}$$

where $MTTF$ = mean time to failure of the system

$MTTR$ = mean time to repair of the system

If both failure and repair times follow the exponential distribution, the availability can be converted into an expression which is independent of time:

$$\tau = \frac{1}{MTTF} = failure\ rate$$

$$\mu = \frac{1}{MTTR} = repair\ rate$$

and

$$\lim_{t \to \infty} A\ (t) = A = \frac{\mu}{\tau + \mu}$$

In this model the following assumptions are made:

(a) The system is balanced. All machines work at the same rate.

(b) Machines have two states, up and down, each with exponential sojourn time. The up and down times variables are statistically independent.

(c) If two machines have failed, repair work can be done on both simultaneously.

(d) Failures are state-dependent, i.e. machines cannot break down when blocked or starved.

(e) The first machine cannot be starved, and the last machine cannot be blocked.

3. THE TWO-MACHINE SYSTEM ONE-BUFFER SYSTEM

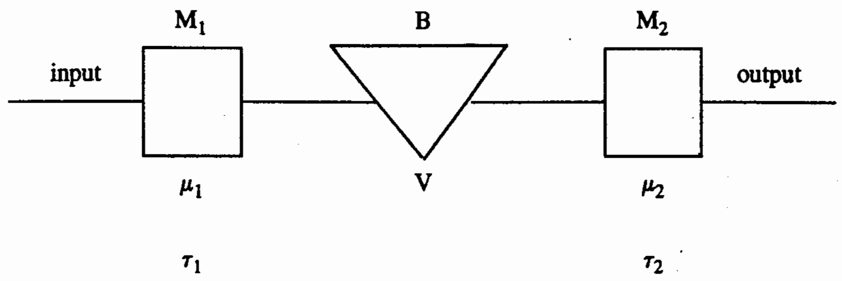

Figure 1. Two-machine configuration

The analytical result for the availability of the two-machine system A_t as obtained by MALATHRONAS et al. [1983] is:

$$A_t = \frac{(p_1 - p_2 e^{-k})\ A_1 A_2}{p_1 A_2 - p_2 A_1 e^{-k}}$$

where

$$p_i = \frac{\tau_i}{\mu_i}, \qquad i = 1,2$$

A_i = *availability of machine i*

$$k = \frac{(\mu_1 + \mu_2 + \tau_1 + \tau_2)\ (\tau_1 \mu_2 - \tau_2 \mu_1)}{(\mu_1 + \mu_2)\ (\tau_1 + \tau_2)\ c_1}$$

V = *buffer size*

Q = *production rate of the system*

$$c_1 = \frac{Q}{V}$$

= *the relative rate at which the buffer is built up or depleted if only one machine is working*

4. THE THREE-MACHINE TWO-BUFFER SYSTEM

The system in which three machines are in series can be depicted as in figure 2. The machines are identified as M_i (i=1..3). The buffers B_1 and B_2 can have different volumes V_1 and V_2. The main idea is to replace the subsystem M_1-B_1-M_2. by an equivalent machine, M_e.

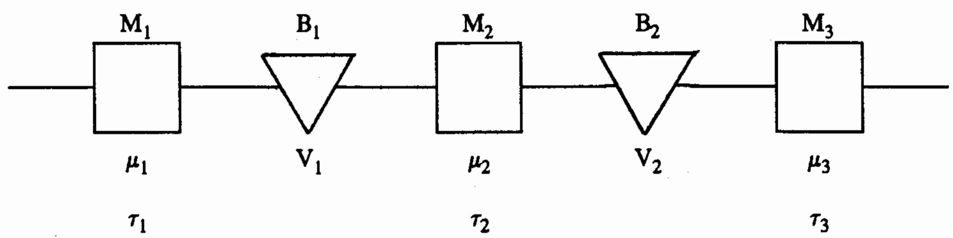

Figure 2: Three-machine two-buffer configuration

Special cases of the three-machine two-buffer system

Some special cases of the system under study can be observed. DE KOSTER and WIJNGAARD [1986] mention the cases in which two machines are unreliable and one machine is perfect. Consider the three possible cases:

(1) first machine is perfect

The inventory.level in B_1 will increase montonically until it reaches the level V_1. the system then will behave as a system consisting of machines 2 and 3 only, separated by the buffer B_2.

(2) second machine is perfect

The state of the system can be described by: (a) the state of machine 1, (b) the state of machine 3, and (c) $u \in [0, V_1+V_2]$ being the sum of the inventory levels of both buffers. Starvation of the third machine is only possible if $u = 0$.

(3) third machine is perfect

If the system starts with an empty buffer V_2, it will always remain empty. Otherwise it will decrease monotonically towards the zero level. The system then behaves as a system consisting of machine 1, buffer B_1 and machine 2.

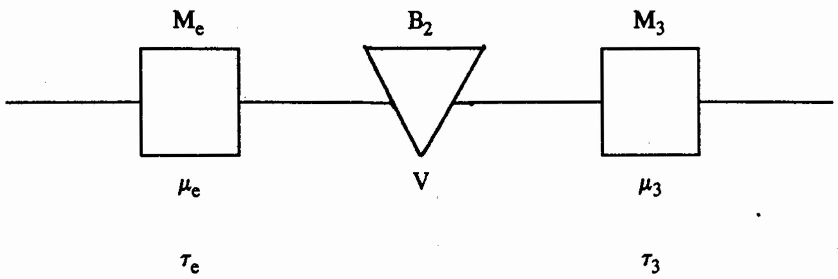

Figure 3: Equivalent machine configuration

The proposed approximation for determining the availability consists in utilizing twice Malathronas' formula (3). The first step is to replace the subsystem M_1-B_1-M_2 by an equivalent machine, M_e. The second step is to analyse the M_e-B_2-M_3 system. By applying Malathronas' formula a second time, it is possible to determine the availability of the entire system. The main difficulty in this approach is that the equivalent machine, M_e, should behave as a single machine, i.e. the equivalent time between failures and the equivalent repair time should follow exponential distributions and should be independent as well of each other as in time.

A more elaborate view on the distribution of both uptimes and downtimes requires the use of a test experiment. In a survey by ASCHER [1990] the test for exponentiality proposed by COX and OAKES [1984] appears to be the best. From simulated results, it appears that M_e's distributions are at least approximately exponential. Details on the test and the results can be found in appendices 1 and 2.

In order to develop a discrete event simulation model for the three-machine two-buffer system ten events are to be taken into consideration: machine 1 goes down (A) or goes up (B), machine 2 goes down (C) or goes up (D), machine 3 goes down (E) or goes up (F), tank 1 becomes empty (G) or full (H), and tank 2 becomes empty (I) or full (J).
In appendix 3 we enumerate the 32 possible system states. The discrete event simulation model can be used for calculation of the steady-state probabilities in each of the states.

4.1. AVAILABILITY OF THE THREE-MACHINE TWO-BUFFER SYSTEM

Similarly as in the two-machine case, the availability of the equivalent machine A_e can be written in terms of τ_e and μ_e as:

$$A_e = \frac{(p_1 - p_2 e^{-k})\, A_1 A_2}{p_1 A_2 - p_2 A_1 e^{-k}} = \frac{\mu_e}{\mu_e + \tau_e} \tag{3}$$

This leaves us with finding analytical expressions for the values of τ_e and μ_e. To calculate μ_e , we first define

$$\begin{aligned} A_e &= \text{Prob}(\ M_2 \text{ up \& not starved }) \\ &= \text{Prob}(\ M_2 \text{ up }) - \text{Prob}(\ M_2 \text{ up \& starved }) \\ &= A_2 - \text{Prob}(\ M_2 \text{ up }) * \text{Prob}(\ M_2 \text{ starved }) \\ &= A_2 - A_2 * \text{Prob}(\ M_2 \text{ starved }) \end{aligned}$$

Further we know that

$$\begin{aligned} &\text{Prob}(\ M_2 \text{ starved }) = 1 - A_e / A_2 \\ &\text{Prob}(\ M_2 \text{ not starved }) = A_e / A_2 \\ &\text{Prob}(\ M_2 \text{ not starved \& down }) = (\ A_e / A_2\ ) * (1 - A_2\ ) \end{aligned}$$

The equivalent repair rate μ_e is the weighted average of the repair rates μ_1 and μ_2

$$\mu_e = \frac{(1 - \frac{A_e}{A_2}) * \mu_1 + (\frac{A_e}{A_2}) * (1 - A_2) * \mu_2}{(1 - \frac{A_e}{A_2}) + (\frac{A_e}{A_2}) * (1 - A_2)}) \tag{4}$$

The equivalent failure rate τ_e can be determined as:

$$\tau_e = \frac{(1 - A_e) * \mu_e}{A_e} \tag{5}$$

4.2. DESIGN AND RESULTS OF A SIMULATION EXPERIMENT

To test the validity of the above results in an empirical way, 24 different cases of two-machine one-buffer systems are calculated using the equations (4) and (5). The numerical results are compared with results obtained by simulation (Table 1). Deviations appear to be very small and within an acceptable range.

The simulation model identifies three entities in the system (Machine 1, Machine 2 and the Buffer) which can be in one of different states. Both machines can be either *up* or *down*. The buffer can be *empty, full* or *at an intermediate level*. This should lead to a system space with twelve states. Similarly as in the analytical approach by MALATHRONAS et al. [1983] we ignore some possible states: (1) the state where M_2 is *down* and the buffer tank is *empty*, since this case only arises if M_2 went down at the precise moment the tank became empty; (2) the state where M_1 is *down* and the tank is *full* for a similar reason.

This leaves us with the following eight states:

1. Both machines down; tank at intermediate level
2. M_1 down, M_2 up ; tank at intermediate level
3. M_1 up, M_2 down ; tank at intermediate level
4. M_1 up, M_2 up ; tank at intermediate level
5. M_1 down, M_2 up ; tank empty (M_2 shut down)
6. M_1 up, M_2 up ; tank empty
7. M_1 up, M_2 down ; tank full (M_1 shut down)
8. M_1 up, M_2 up ; tank full.

While in general the results of this simplifying approximation is very promising, it is interesting to investigate why some results are worse than others. This can lead to some advice or warning for the user of this approximation.

TABLE 1. Comparison between analytical and simulated results

CASE NO.	MTTF		MTTR			by FORMULA		by SIMULATION	
	M1	M2	M1	M2	V	UPTIME	DOWN-TIME	UPTIME	DOWN-TIME
1	20	30	10	20	45	23.14	17.71	22.87	17.73
2	40	30	50	20	25	19.63	30.37	19.49	30.47
3	40	80	40	100	40	37.22	67.92	37.22	67.12
4	45	55	30	35	55	36.02	33.28	35.39	33.04
5	55	56	10	10	20	38.04	10.00	37.98	10.06
6	55	60	10	15	25	43.06	13.59	42.85	13.66
7	55	60	15	5	20	33.23	9.46	33.05	9.68
8	55	60	15	15	15	35.56	15.00	35.29	15.14
9	55	60	45	50	35	36.41	48.03	36.04	47.94
10	55	60	80	10	20	30.00	45.00	29.80	46.02
11	55	60	80	10	120	30.17	43.23	30.37	43.83
12	55	60	80	10	200	30.52	44.39	30.31	45.53
13	55	60	80	15	30	30.76	46.68	30.98	46.36
14	60	50	10	20	20	39.33	17.87	39.08	18.11
15	60	55	10	80	20	43.98	65.98	43.79	66.68
16	60	55	10	80	120	54.23	78.00	54.64	78.12
17	60	90	70	75	80	48.69	72.71	48.46	72.02
18	60	95	10	12	40	56.82	11.20	56.85	11.27
19	70	80	65	60	50	46.79	62.08	46.99	61.99
20	75	20	10	20	10	18.32	19.16	18.23	19.19
21	80	60	10	5	40	45.85	6.18	45.64	6.19
22	85	100	20	100	40	72.49	77.99	72.33	78.55
23	90	60	12	10	30	49.28	10.36	48.88	10.44
24	100	50	60	25	45	38.68	32.92	38.29	32.99

1. All values are expressed in time units.
2. The length of the simulation runs is 10,000 time units which is sufficient to reach a reasonably steady state situation.

A first observation concerns whether the worse of the approximated mean times is either the uptime or the downtime period (or both considered equal if the difference is less than 0.1% . If we split the sample in two parts, a comparison between the 'best' performing and the 'worst' performing half gives us the following table:

	WORSE	BEST
UPTIME	3	5
EQUAL	0	3
DOWNTIME	9	4

This assures us that, if problems with the approximation appear, they are mainly due to the estimation of the mean downtime.

While investigating the cause for this effect, a second observation can be made. Let us compare the deviation from the exponential distribution (measured in terms of the variation coefficient) for both uptime and downtime. Averaged over 24 samples the variation coefficients for uptime and downtime are resp. 0.99 and 1.14. According to this measure we can conclude that uptimes are nearly exponential, but downtimes are not. This motivates us to take a closer look at how the variation coefficient can affect the 'worse' performance of the downtime.

The Cox and Oakes test statistic shows no deviation from exponentiality for the uptime distribution. Significant deviations appear for the downtime distribution. In all significant deviations the coefficient of variation is larger than one.

This leads indeed to an indication that a degration of the exponentiality of the empirical downtime distribution can be a cause of error in the approximation.

After determining μ_e and τ_e , the next step is to use formula (3) once again, resulting in the following expression of overall system availability:

$$A_t = \frac{(p' - p_3 e^{-k}) A_e A_3}{p' A_3 - p_3 A_e e^{-k'}} \tag{6}$$

where, $p' = \frac{\tau_e}{\mu_e}$

$p_3 = \frac{\tau_3}{\mu_3}$

A_3 = *availability of* M_3

$k' = \frac{(\mu_e + \mu_3 + \tau_e + \tau_3)(\tau_e\mu_3 - \tau_3\mu_e)}{(\mu_e + \mu_3)(\tau_e + \tau_3) c_2}$

$c_2 = \frac{Q}{V_2}$

A_e = *availability of subsystem* $M_1 - B_1 - M_2$

4.3. OPTIMAL SIZES OF THE BUFFERS

Optimal buffer sizes are determined in two phases. In a first step a decision is made on the size of the total buffer. It is assumed that both types of buffers are comparable: they compete for the same storage space and they incur the same cost per volume unit. In a second step a decision is made on the partitioning of the total buffer size.

In the proposed procedure the total buffer size T is either determined by a restriction on the total buffer space or by considering the trade-off between increase in availability and increase in storage costs. Once T is fixed the proportions of T allocated to the first buffer and to the second buffer can be determined by maximizing formula (6) in which V_2 has been replaced by T-V_1. This optimization problem is non-linear in one variable V_1, the size of the first buffer. The optimum can be determined by a one-dimensional grid search.

If the total buffer size T is not limited by a constraint on storage space, T should be so large that any further increase of T has no significant effect on the total availability A_t of the system. This value of T is determined by

$$\frac{dA_t}{dT} = c$$

where c is calculated from

(Profit) * (Production rate) * dA_t = (Storage cost) * dT

$$c = \frac{(storage\ cost)}{(profit)\ *\ (productionrate)}$$

In practice a series of increasing values of T are considered together with their corresponding optimal availability values A_t. For determining the optimal availability, every time a one-dimensional grid search should be made in order to calculate the optimal partitioning of T. Increasing values of T correspond with increasing values of A_t. However, the increase of A_t becomes insignificant when $\Delta A_t < c * \Delta T$, and the procedure should be stopped.

As an *example* we consider the following system:

$\tau_1 = 0.03$, $\tau_2 = 0.08$, $\tau_3 = 0.02$;
$\mu_1 = 0.1$, $\mu_2 = 0.5$, $\mu_3 = 0.15$.

The fill rates c_1 and c_2 are chosen in such a way that the buffers can be emptied (if full) or filled (if empty) in a unit of time. This means if the buffer size is 80, the fill rate takes the value 1/80. Total buffer sizes take values 80, 60, 40, 30 and 20. Figure 4 show the availability of the total system in function of the first buffer size (holding the total buffer size constant). If e.g. the total buffer size of 80 has to be divided in a first buffer size of 60 and a second one of 20, the values $c_1 = 1/60$ and $c_2 = 1/20$ should be used in the formulae above.

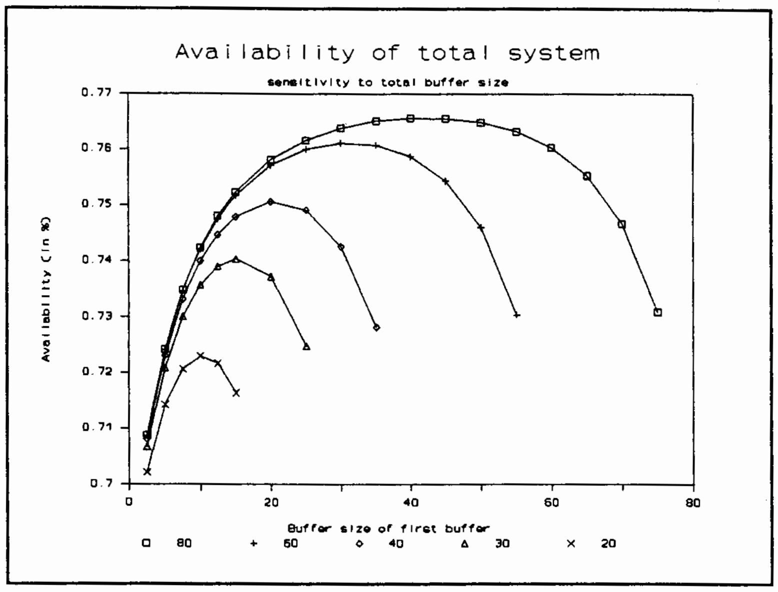

Figure 4: Availability of the total system in function of buffer assignments

CONCLUSION

The above approach is very simple and is valid unless the basic assumptions are not satisfied. The same procedure can be repeated to analyze multi-stage systems with more than three machines. It may be that the exponential properties of the equivalent machines will deteriorate. However, the approach will probably still provide good approximate values for availability calculations.

In practice, several adjacent machines can often be grouped together heuristically and represented by one machine such that the operational characteristics of this single machine are almost indentical with the real configuration. This possibility certainly enlarges the applicability of the results obtained for a three-machine system.

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Appendix 1 : The Cox and Oakes test statistic

Let the observations of a sample of size N have values x_i (i=1,2,...,N) and let

$$\hat{\rho} = \frac{N}{\sum x_i}$$

Compute the following variables:

$$U = N + \sum \log x_i - N\frac{\sum x_i \log x_i}{\sum x_i}$$

$$I_{KK} = N + \sum (\hat{\rho}\, x_i)\, [\log (\hat{\rho}\, x_i)]^2$$

$$I_{K\rho} = \sum x_i \log (\hat{\rho}\, x_i)$$

$$I_{\rho\rho} = N/\hat{\rho}^2$$

$$\nu_{KK} = (I_{KK} - I^2_{K\rho} / I_{\rho\rho})^{-1}$$

The signed statistic

$$U \sqrt{\nu_{KK}}$$

is approximately distributed as a standard normal deviate on the null hypothesis. The null hypothesis states that the sample is drawn from an exponential distribution.

Appendix 2. Table with results on the Cox & Oakes statistic for exponentiality

CASE NO.	MTTF		MTTR			Number of observations, Cox & Oakes statistic & Coefficient of variation					
	M1	M2	M1	M2		FAILURE			REPAIR		
					BUFFER	N° of observ	Cox & Oakes	Coeff. of var.	N° of observ	Cox & Oakes	Coeff. of var.
1	20	30	10	20	45	258	-1,049	0,97	258	-1,006	1,07
2	40	30	50	20	25	202	1,189	0,95	201	-0,203	1,05
3	40	80	40	100	40	93	0,440	0,95	92	**-2,917**	1,41
4	45	55	30	35	55	141	1,067	0,94	140	-1,009	0,99
5	55	56	10	10	20	206	0,232	0,96	205	0,107	1,01
6	55	60	10	15	25	162	1,129	0,99	162	-0,166	0,98
7	55	60	15	5	20	263	-1,801	1,09	263	**-2,511**	1,09
8	55	60	15	15	15	199	1,866	0,93	199	-1,616	1,11
9	55	60	45	50	35	122	0,534	0,98	122	-1,062	1,19
10	55	60	80	10	20	134	1,228	0,88	134	**-6,659**	1,71
11	55	60	80	10	120	123	0,843	0,90	123	**-5,536**	1,39
12	55	60	80	10	200	144	-1,245	1,07	143	**-6,302**	1,48
13	55	60	80	15	30	131	0,385	0,95	131	**-5,272**	1,44
14	60	50	10	20	20	186	0,640	0,95	186	-0,002	0,97
15	60	55	10	80	20	85	0,745	1,00	84	-1,143	1,04
16	60	55	10	80	120	73	0,561	0,96	72	-0,826	1,07
17	60	90	70	75	80	79	1,205	0,90	79	-0,898	0,98
18	60	95	10	12	40	141	-0,489	1,01	141	0,398	0,92
19	70	80	65	60	50	102	-1,477	1,19	101	-1,192	1,07
20	75	20	10	20	10	282	-0,239	1,03	281	-1,098	1,04
21	80	60	10	5	40	193	0,042	0,96	193	-0,520	0,97
22	85	100	20	100	40	58	0,835	0,87	57	0,204	0,89
23	90	60	12	10	30	169	0,545	0,98	169	0,013	1,04
24	100	50	60	25	45	148	1,881	0,88	148	**-2,391**	1,31

Bold printed values exceed the critical bounds of a standard normal deviate. (95%)
For these cases we do not find empirical evidence that the repair (or failure) times are exponentially distributed.

Appendix 3: Enumeration of the three-machine two-buffer system state space

1.	down	down	down	$0 < Y1 < vol\ 1$	$0 < Y2 < vol\ 2$
2.	down	down	up	$0 < Y1 < vol\ 1$	$0 < Y2 < vol\ 2$
3.	down	up	down	$0 < Y1 < vol\ 1$	$0 < Y2 < vol\ 2$
4.	down	up	up	$0 < Y1 < vol\ 1$	$0 < Y2 < vol\ 2$
5.	up	down	down	$0 < Y1 < vol\ 1$	$0 < Y2 < vol\ 2$
6.	up	down	up	$0 < Y1 < vol\ 1$	$0 < Y2 < vol\ 2$
7.	up	up	down	$0 < Y1 < vol\ 1$	$0 < Y2 < vol\ 2$
8.	up	up	up	$0 < Y1 < vol\ 1$	$0 < Y2 < vol\ 2$
9.	down	up	down	$Y1 = 0$	$0 < Y2 < vol\ 2$
10.	down	up	up	$Y1 = 0$	$0 < Y2 < vol\ 2$
11.	up	up	down	$Y1 = 0$	$0 < Y2 < vol\ 2$
12.	up	up	up	$Y1 = 0$	$0 < Y2 < vol\ 2$
13.	down	down	up	$0 < Y1 < vol\ 1$	$Y2 = 0$
14.	down	up	up	$0 < Y1 < vol\ 1$	$Y2 = 0$
15.	up	down	up	$0 < Y1 < vol\ 1$	$Y2 = 0$
16.	up	up	up	$0 < Y1 < vol\ 1$	$Y2 = 0$
17.	up	down	down	$Y1 = vol\ 1$	$0 < Y2 < vol\ 2$
18.	up	down	up	$Y1 = vol\ 1$	$0 < Y2 < vol\ 2$
19.	up	up	down	$Y1 = vol\ 1$	$0 < Y2 < vol\ 2$
20.	up	up	up	$Y1 = vol\ 1$	$0 < Y2 < vol\ 2$
21.	down	up	down	$0 < Y1 < vol\ 1$	$Y2 = vol\ 2$
22.	down	up	up	$0 < Y1 < vol\ 1$	$Y2 = vol\ 2$
23.	up	up	down	$0 < Y1 < vol\ 1$	$Y2 = vol\ 2$
24.	up	up	up	$0 < Y1 < vol\ 1$	$Y2 = vol\ 2$
25.	down	up	up	$Y1 = 0$	$Y2 = 0$
26.	up	up	up	$Y1 = 0$	$Y2 = 0$
27.	down	up	down	$Y1 = 0$	$Y2 = vol\ 2$
28.	up	up	down	$Y1 = 0$	$Y2 = vol\ 2$
29.	up	up	down	$Y1 = vol\ 1$	$Y2 = vol\ 2$
30.	up	up	down	$Y1 = vol\ 1$	$Y2 = vol\ 2$
31.	up	down	up	$Y1 = vol\ 1$	$Y2 = 0$
32.	up	up	up	$Y1 = vol\ 1$	$Y2 = 0$

The first tandem of machines can exist in a set of eight states according to MALATHRONAS et al. [1983]. These states can be combined with the second buffer at intermediate level and the third machine either up or down. This leads to 16 states in the total system (states 1 to 12 and 17 to 20 in the table above).

Similar to the reason why states in the two-machine one-buffer system are neglected, also here it can be stated that the combination of the second buffer being empty and the third machine down does not exist. This leaves us with the state (Buffer 2 empty, Machine 3 up) combined with the eight states of the first tandem. This adds 8 states to the total system (states 13 to 16, 25 to 26 and 31 to 32 in the table above).

Similarly the second buffer being full and machine 3 being up does not exist. This leaves us with the state (Buffer 2 full, Machine 3 down) combined with the eight states of the first tandem. This adds 8 states to the total system (states 21 to 24, 27 to 30).